Growth with offshoring

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Abstract

I discussed innovation and offshoring in a semi-endogenous model to study the role of offshoring and impacts of offshoring cost reduction. I analyzed how labor reallocation among R&D sector and production sector influence the innovation activity and welfare. Offshoring makes both N and S to exploit their comparative advantage in the aspect of innovation and production. I identified a save labor effect and a relative demand effect which drive the economy. (JEL: F1, F23, F43, F6)

1 Introduction

The technology developments changes the nature of trade and production. As in Baldwin(2012), ICT (Information and communications technology) makes global supply chain possible and wage gap makes it profitable. On the one hand, more and more big firms are involved in Global Supply Chain (GSC) production. One famous example is the Apple. While keeping the innovation stage in U.S., most part of the manufacturing and assembly is done in Asia. Though there are hot debates about the moral issue of working conditions in Asia, its impacts on unemployment in U.S., we cannot deny that the company makes profits and succeed. On the other hand, economists nowadays frequently argued that the economy promotion, or growth miracle in some one's description, in LDCs are attributed to GSC. LDCs now can benefit from DCs and grow by joining the GSC even though they don't have the technology of the final good in the supply chain.

In this paper, I developed models based on Product Circle Model with semi-endogenous growth in autarky, and with offshoring. In section 2, I present the models and BGP e-quilibria. To make distinction between offshoring and trade of goods, I assume that parts of new goods cannot be replaced by old goods. This is a quite realistic assumption since most goods offshore in real world is high-tech ones. For example, many LED (liquid-crystal display) companies produce final goods, like TV monitor, phones screen in less developed countries (LDC) and remain the key technology of how to produce LED chips in the headquarter. But for LDCs, how to produce monitors and screens from LED chips is new to them as well. One important implication of this assumption is that by doing offshoring, S will gain some technology benefits from N because N need to "teach" S how to do those parts. On aggregate, it seems like that S is more efficient in doing R&D, producing more innovation with the same amount of investment. But the reason lies behind this is the direct technology increment by doing offshoring.

Following Jones(1995), I consider different cases of innovation externalities. First, existing innovations have externality on future innovations. The stock of knowledge can make it easier to do research as major innovations raise the productivity of the following scientists (stand on the giant's shoulder). Alternatively, perhaps easier innovations are developed first and at a high level of social knowledge, it is more difficult to discover new things (fishing-out). Second, existing researchers have externality. Or more pre-cisely, innovation might be DRS or IRS to the input researchers. To save terminology, I refer to this as researcher externality. For example, team work is more productive and some research cannot be done by a small amount of people. In contrast, there might be duplicate research. As said in Jones(1995), Romer(1990), it is more like a philosophy question whether the externalities are positive or negative. All the results depends on these two externalities.

In section 3, I discuss the main mechanisms how offshoring affects N and S. Offshoring affects labor allocation, innovation, consumption and wage in both N and S. The main mechanisms in my paper are labor reallocation and changes in relative demand. Offshoring frees some labor in N from production and the ratio of R&D workforce to production workforece increases in N while decreases in S. This will propel the new sector and increase the number of innovations if the externality is positive. S benefits from offshoring by an additional way: the direct technology diffusion. Since consumption and wage rate in N and S depend on the number of innovations, they changes accordingly. Result will be the opposite with negative externality.

A more subtle but interesting channel in my model is through the changes in markup of new goods and thus the relative demand between new goods and old goods. The intuition is very simple. As parts of the new goods are produced in a cheaper way — produced in S, the price of it is cut down. This will increase the demand of new goods and promotes the new sector in return. This force also affects consumption and wage in N directly because the wage worth more with offshoring and thus the welfare is improved.

Information and communication technology changes rapidly. Phones, video conferences, emails, even remote control of machines are very common now. Consequently, companies can perform offshoring more and more easily. It is nature to ask the question: what's the impact of reduction in offshoring cost? The existing papers arrive on uncertain conclusions on wage gaps. Costinot, Vogel and Wang (2012) found that changes in wage gaps of top production level countries and bottom ones differ. Grossman and Rossi-Hansberg (2008) found that reductions in offshoring have both positive and negative effects on wage gap. After analyzing the impacts of offshoring, I also studied the results of reduction in offshoring cost in section 4. Similarly, I also found opposite forces affect the wage gap. Let's consider the negative externality case. The direct changes in offshoring share due to cost reduction will narrow the wage gap because of the changes in labor demand. It increases the demand for offshoring labor in S while reduce the demand in N. So the wage gap between S and N decreases. However, reduction in offshoring costs will also change the relative demand between old goods and new goods in N. If the new sector expands, both wage rates in N and S increase. Which country benefits more from the cost reduction depends on current offshoring share and properties of innovation process in N. For other externality parameters, results will be different. But the two opposite forces remain. The desire to offshore more drives wage rate in N and S oppositely while the relative demand for new goods moves wage rate in N and S in the same direction. Whether the new sector expands or not in reaction to cost reduction depends on the current cost level. It is discussed below.

Cost reduction will affect innovation and welfare as well. Labor reallocation is again the main mechanism behind. Consider the case of negative externality. As cost decreases, the desire of more offshoring pushes more workers in N into the R&D sector and improves welfare. If the cost reduction lowers down the mark-up of new goods and thus the new sector expands, it will demand more labor and thus increases the number of innovation. Also, the smaller mark-up increases consumers' buying power in N (their wages worth more). All the forces improves welfare in N and S. For other externality parameters, results will be different. I will discuss the results in detail in the paper. Again, how the mark-up and the new sector reacts to cost reduction depends on the current cost level and is discussed below.

One important feature of offshoring cost in this paper is the increasing marginal change. Cost reduction means the average cost reduces. However, the marginal cost decreases nonlinearly. When offshoring cost is high, reducing cost will lead to a lower marginal cost. When cost is already very low, the marginal cost increases as average cost decreases. This is very realistic. It can be support by the fact that easy parts are offshored first and followed by parts demanding skill. When offshoring cost is high, companies offshore the tasks that doesn't need much skill, such as packaging, sewing and cutting. Then if the transport and management cost lowers down, it will be cheaper to offshore an additional unit of easy tasks. However, when the cost is already very low, companies will offshore a lot and the further reduction in costs can only prompt complex tasks to be offshored, like operating the machine, assembly electric components etc. Difficult tasks demand a lot of training, organization and management, and thus an increasing amount of extra cost.

This feature has important implications on mark-up of new goods and thus the relative demand. The mark-up depends not only on the total offshoring cost but also on the marginal cost. When the reduction in total cost exceeds the marginal increasing (or total cost and marginal cost both decrease), cost reduction lowers down the mark-up and prompts the demand of new goods. Thus the new sector expands. In contrast, when the reduction in total cost is outweighed by the marginal increment, cost reduction enlarges the mark-up and decreases the demand of new goods. Thus the new sector shrinks. An implication is that if we consider the relative demand effect, cost reduction is not beneficial when already cost is very low.

To sum up, the main contributions of my paper are the followings. First, I discussed the impacts of GSC in a semi-growth model and derive welfare implications from its influence on innovations. Consistent with existing result, I find that in most cases, offshoring has positive impacts. However, as I consider the question in a semi-growth model, it emphasizes the role of innovation externality. When the externality of existing innovations and researchers are high, offshoring will have negative effects, especially to N. Second, I studied the general equilibrium in which wages are determined endogenously. This brings out the relative demand effect. Offshoring will affect the cost of new products, and thus their price. As a result the demand and people's buying power change. This will affect the economy in two ways. One is the demand's impact on the new sector and the other is the direct welfare improvement due to a lower mark-up. Third, R&D activities are important in my model. Consequently, labor allocation between R&D sector and production sector reacts to offshoring and changes in offshoring cost. Innovation and welfare also depends on the labor allocation. Finally, I introduced an interesting measurement of offshoring cost changes. It allows heterogenous costs of different parts. It implies that cost reduction is more beneficial when cost is high if easy parts are offshored first.

The paper is organized in the following way. Section 2 presents the model in autarky and with offshoring. Section 3 discuss how offshoring affects N and S in detail. In section 4, I show different channels of offshoring costs' impacts.

2 The Model

In this section, I present the models for autarky, free trade between countries and autarky with offshoring. For the autarky economy, I use the standard endogenous growth model with an elimination the "scale effect" as Jones(1995).

There are two kind of countries, the North (N) and the South (S). N represents the advanced countries while S represents the less advanced countries. The main differences in my paper between N and S are wage rates and innovation abilities. S has a low real wage while N is better at innovation. I will discuss the difference in detail in each scenario.

2.1 Autarky

In this section, I present the autarky economies in N and S. In each time period, there is one final good and $N(t)$ types of intermediate goods. The final good is combined by different types of intermediate goods costlessly in the Dixit-Stiglitz(1977) way: $C_j(t) = (\int_0^{N(t)} c_j(i,t)^{\frac{\epsilon-1}{\epsilon}} \mathrm{d}i)^{\frac{\epsilon}{\epsilon-1}}$. Consumers' utility is increasing in the amount of consumption of final good. Consumers have the same utility function in North and South. Following the usual convention, I assume that a represented consumer maximizes an additively separable utility function subject to the dynamic budget constraint:

$$\max_{c_j(t)} \int_0^{\infty} e^{-\rho t} \frac{C_j(t)^{1-\theta} - 1}{1-\theta} \mathrm{d}t$$

$$s.t. C_j(t) = (\int_0^{N(t)} c_j^v(i,t)^{\frac{\epsilon-1}{\epsilon}} \mathrm{d}i)^{\frac{\epsilon}{\epsilon-1}}$$

$$\int_0^{N(t)} c_j(t,i) p(t,i) \mathrm{d}i = P_j(t) C_j(t) = I_j(t)$$

$$\dot{B}_j(t) = r_j(t) B_j(t) + Y_j(t) - C_j(t)$$

where, j is the country index, for now $j = N, S$, v is the goods index, for N, $j = n, o$, that is there are new goods and old goods. For S, $j = o$, that is there is only old goods. $c_j(t) = \frac{C_j(t)}{L_j(t)}$ is the per capita consumption of final good. $c_j^v(i,t)$ is the aggregate amount of type i intermediate good. $I_j(t)$ is the income in j.

We the Dixit-Stiglitz price index $P_j(t) = (\int_0^{N(t)} p(t,i)^{1-\epsilon} \mathrm{d}i)^{\frac{1}{1-\epsilon}}$. So we can have the demand functions that are,

$$c_j^v(t,i) = (\frac{p_j^v(i,t)}{P_j(t)})^{-\epsilon} C_j(t) = p_j^v(i,t)^{-\epsilon} I_j(t) P_j(t)^{\epsilon-1} \quad (1)$$

From the Euler Equation, we have that the per capita consumption growth rate follows

$$g_{c_j} = \frac{1}{\theta} (r_j(t) - \rho - n) \quad (2)$$

.

There are two kind of intermediate goods, the new ones and the old ones. N can produce both new goods and old goods. Firms have monopoly power in new sector but are perfect competitive in old sector. In S, they can only produce old goods and they have monopoly power.

One unit of intermediate good is produced by one unit of labor. For firms in perfect competitive sector, they are price takers. The market drives their price to the marginal cost: the wage rate. For firms with monopoly power, it faces a downward-sloping demand (1) and can choose the price to maximize its profits. That is, they solves the following problem every period,

$$\max_{p_j^v(i,t)} (p_j^v(i,t) - w_j) c_j^v(i,t) \tag{3}$$

$$s.t. \quad c_j^v(i,t) = (\frac{p_j^v(i,t)}{P_j(t)})^{-\epsilon} C_j(t)$$

where w_j is the real wage in country j. The result is a standard monopoly problem with constant marginal cost and constant elasticity of demand, and it is readily solved to yield the following equations. Price is a constant mark-up of wage and profit is positive.

$$p_j^v(i,t) = \frac{\epsilon}{\epsilon - 1} w_j(t) \tag{4}$$

$$\pi_j^v(i,t) = \frac{1}{\epsilon - 1} w_j(t) c_j^v(i,t) \tag{5}$$

So all intermediate goods within one type (new goods in N, old goods in N and old goods in S) have the same price and thus from (1), have the same demand. For below, I will omit i and use the total demand for one type of intermediate goods as $N_j^v(t) c_j^v(t)$.

I adopt the innovation process in Jones(1995) to include the externality of existent innovation and duplicate researches. However, there is a difference between N and S.

N innovates in new goods while S does it old ones. For N, the number of old goods increases due to the diffusion of technology in new ones. This is consistent of firms' behavior described above. So the innovation processes in N and S are the followings respectively.

In N,

$$\dot{N}_N^n(t) = \eta \left(L_N^R(t) \right)^\lambda N_N^n(t)^\phi \tag{6}$$

$$\dot{N}_N^o(t) = \eta \left(L_N^R(t) \right)^\lambda N_N^o(t)^\phi, \tag{7}$$

$$\dot{N}_N(t) = \eta \left(L_N^R(t) \right)^\lambda N_N^n(t)^\phi + \eta \left(L_N^R(t) \right)^\lambda N_N^o(t)^\phi. \tag{8}$$

In S,

$$\dot{N}_S^o(t) = \eta \left(L_S^R(t) \right)^\gamma N_S^o(t)^\phi. \tag{9}$$

N is more efficient in doing R&D in that λ is bigger than γ.

The value of innovation $V_j(t)$ is determined by the non-arbitrage condition. The holder of the innovation is indifferent between selling it to the market or keeping it to general monopoly rents.

$$r_j(t)V_j(t) = \pi_j(t) + \dot{V}_j(t) \tag{10}$$

The free entry of R&D sector makes a worker indifferent between being in production sector to get the wage or be in R&D sector. So the value of innovation satisfies,

$$V_N(t)\frac{\dot{N}_j^v(t)}{L_j^R(t)} = V_N(t)\eta \left(L_j^R(t) \right)^{\lambda-1} N_j^v(t)^\phi = w_j(t) \tag{11}$$

Combine (5)(10)(11), we get,

$$\frac{w_j(t)}{\eta \left(L_j^R(t) \right)^{\lambda-1} N_j^v(t)^\phi} = \frac{1}{\epsilon - 1} \int_t^\infty e^{-\int_0^x r(x)dx} w_j(s) c_j^v(s) ds \tag{12}$$

(in(11) (12), for $j = S$, λ should be replaced by γ.)

To recap, The distinctions between S and N lie in three aspects. First, real wage rate in S is lower than that in N ($w_S < w_N$). Second, there are both new and old goods in N while only old ones in S. Moreover, S innovates in old goods while N innovate in new ones. Third, N is more efficient in innovation than S. Other parameters are equal between N and S, including discount rate, population growth rate.

We normalize price index in N to 1.

Now we can define the autarky equilibria in each country separately.

Definition 1. *An autarky equilibrium in N is a set of sequences of consumptions $\{c_N^v(t)\}$, sequences of prices and wage rate $\{p_N^v(t)\}$, $\{w_N(t)\}$, sequences of labor supply $\{L_N^n(t)\}$, $\{L_N^o(t)\}$,$\{L_N^R(t)\}$, and the sequences of number of new and old goods $\{N_N^v(t)\}$(where $v = n, o$), which is determined by the following conditions.*

(a) The represented household optimize their consumptions. So demand function follows (1), that is

$$c_N^v(t) = (\frac{p_N^v(t)}{P_N(t)})^{-\epsilon} C_N(t) = p_N^v(t)^{-\epsilon} I_N(t) P_N(t)^{\epsilon-1}.$$

$$I_N(t) = w_N(t) L_N(t)$$

(b) Firms in new sector maximize their profits while firms in old sector are competitive.

$$p_N^n(t) = \frac{\epsilon}{\epsilon - 1} w_N(t).$$

$$p_N^o(t)) = w_N(t),$$

$$P_N(t) = (N_N^n(t) p_N^n(t)^{1-\epsilon} + N_N^o(t) p_N^o(t)^{1-\epsilon})^{\frac{1}{1-\epsilon}} = 1$$

(c) Goods market clears.

$$c_N^n(t) = \frac{L_N^n(t)}{N_N^n(t))},$$

$$c_N^o(t) = \frac{L_N^o(t)}{N_N^o(t))}.$$

(d) Labor market clears.

$$L_N^n(t) + L_N^o(t) + L_N^R(t) = L_N(t).$$

(e) Free entry in R&D sector, as in (12)

$$\frac{w_N(t)}{\eta\left(L_N^R(t)\right)^{\lambda-1} N_N^n(t)^{\phi}} = \frac{1}{\epsilon-1}\int_t^{\infty} e^{-\int_0^x r(x)dx} w_N(s) c_N^n(s) ds$$

(f) Innovation process follow (6)-(8).

$$\dot{N}_N^n(t) = \eta\left(L_N^R(t)\right)^{\lambda} N_N^n(t)^{\phi},$$

$$\dot{N}_N^o(t) = \eta\left(L_N^R(t)\right)^{\lambda} N_N^o(t)^{\phi},$$

$$\dot{N}_N(t) = \eta\left(L_N^R(t)\right)^{\lambda} N_N^n(t)^{\phi} + \eta\left(L_N^R(t)\right)^{\lambda} N_N^o(t)^{\phi}.$$

Definition 2. *An autarky equilibrium in S is a set of sequences of consumptions* $\{c_S^o(t)\}$*, sequences of prices and wage rate* $\{p_S^o(t)\}$*,* $\{w_S(t)\}$*, sequences of labor supply* $\{L_S^o(t)\}$*,*$\{L_S^R(t)\}$*,and the sequences of number of old goods* $\{N_N^o(t)\}$*, which is determined by the following conditions.*

(a) The represented household optimize their consumptions. So demand function follows (1), that is

$$c_S^o(t) = (\frac{p_S^o(t)}{P_S(t)})^{-\epsilon} C_S(t) = p_S^o(t)^{-\epsilon} I_S(t) P_S(t)^{\epsilon-1}.$$

$$I_S(t) = w_S(t) L_S(t)$$

(b) Firms in old sector maximize profits.

$$p_S^o(t) = \frac{\epsilon}{\epsilon - 1} w_S(t).$$

$$P_S(t) = (N_S^o(t) p_S^o(t)^{1-\epsilon})^{\frac{1}{1-\epsilon}}$$

(c) Goods market clears.

$$c_S^o(t) = \frac{L_S^o(t)}{N_S^o(t))}.$$

(d) Labor market clears.

$$L_S^o(t) + L_S^R(t) = L_S(t).$$

(e) Free entry of R & D sector, as in (12)

$$\frac{w_S(t)}{\eta \left(L_S^R(t)\right)^{\gamma-1} N_S^o(t)^{\phi}} = \frac{1}{\epsilon - 1} \int_t^{\infty} e^{-\int_0^x r(x)dx} w_S(s) c_S^o(s) ds$$

(f) Innovation process follow (9).

$$\dot{N}_S^o(t) = \eta \left(L_S^R(t)\right)^{\gamma} N_S^o(t)^{\phi}.$$

2.1.1 BGP Equilibria in autarky

Along the balanced growth path, the growth rate of innovation is constant by definition. Just as in Jones(1995), we can solve for this rate and find that it is determined by exogenous parameters. From the labor market clearing condition (d) in definition 1, we know that $g_{L_N^n} = g_{L_N^o} = g_{L_N^R} = g_{L_N} = n$, where n is the the population growth rate. From (6), we have, $g_{N_N^n} = \lambda n + \phi g_{N_N^n}$. So, the growth rate of new innovations is $g_{N_N^n} = \frac{\lambda n}{1-\phi}$. Similarly, we can get the growth rate of old innovations from (7) which is $g_{N_N^o} = \frac{\lambda n}{1-\phi}$. Then from (8), we know that along the BGP, the growth rate of total innovation is the same as new and old innovations', that is $g_{N_N} = g_{N_N^n} = g_{N_N^o}$. To sum up, we have the following equation.

$$g_{N_N} = g_{N_N^n} = g_{N_N^o} = \frac{\lambda n}{1-\phi} \tag{13}$$

Rewrite (6) and (7), we have $\frac{\dot{N}_N^n(t)}{N_N^n(t)} = \eta \left(L_N^R(t)\right)^{\lambda} N_N^n(t)^{\phi-1}$ and $\frac{\dot{N}_N^o(t)}{N_N^o(t)} = \eta \left(L_N^R(t)\right)^{\lambda} N_N^o(t)^{\phi-1}$. Combining with (13), we have $\frac{\dot{N}_N^n(t)}{N_N^n(t)} = \frac{\dot{N}_N^o(t)}{N_N^o(t)}$, which implies $N_N^n(t) = N_N^o(t)$. Using the demand function (a) and goods market clear condition (c) in definition 1, we have that,

$$L_N^o(t) = \frac{1}{1+\beta} L(t) \tag{14}$$

$$L_N^n(t) = \frac{\beta}{1+\beta} \frac{\epsilon - 1}{\epsilon} L(t) \tag{15}$$

$$L_N^R(t) = \frac{\beta}{1+\beta} \frac{1}{\epsilon} L(t) \tag{16}$$

where $\beta = \left(\frac{\epsilon-1}{\epsilon}\right)^{\epsilon-1}$. From the demand function (a) in definition 1 and definition of aggregate consumption (1) we have the growth rate of total aggregate consumption is ,

$$g_{C_N} = n + \frac{1}{\epsilon - 1} g_{N_N}$$

So the growth rate of per capita aggregate consumption is,

$$g_{c_N} = \frac{1}{\epsilon - 1} g_{N_N} \tag{17}$$

From the price conditions (b) in definition 1, we have the growth rate of wage,

$$g_{w_N} = \frac{1}{\epsilon - 1} g_{N_N} \tag{18}$$

From (2), we have the interest rate $r(t)$ is constant and equals

$$r = \theta g_{c_N} + \rho + n \tag{19}$$

Combine (17) (19), we have,

$$r - n = \frac{\theta}{\epsilon - 1} g_{N_N} + \rho \tag{20}$$

Since every variable grows at constant rate, we can rewrite (12) as

$$\frac{w_N(t)}{\eta \left(L_N^R(t)\right)^{\lambda-1} N_N^n(t)^{\phi}} = \frac{1}{\epsilon - 1} \frac{w_N(t) L_N^n(t)}{N_N^n(t)} \frac{1}{r + g_{N_N} - g_{w_N} - n} \tag{21}$$

Substitute (13) (15) (16) (18) and (20) into (21) and simplify, we have,

$$N_N^n(t)^{1-\phi} = \frac{\eta}{(\epsilon - 1)(\rho + \frac{\theta+\epsilon-2}{\epsilon-1}\frac{\lambda n}{1-\phi})} \left[\frac{1}{\epsilon(1 + \frac{1}{\beta})}\right]^{\lambda} (\epsilon - 1) L_N(t)^{\lambda} \tag{22}$$

To sum up, we have the following result for N. The full derivation is in Appendix.

Result 1. *On the BGP in N,*

(a) The number of intermediate goods is, (all t is omitted and remember P_N is normalized to 1.)

$$N_N = 2N_N^n = 2\left[\frac{\eta}{\rho + \frac{\theta+\epsilon-2}{\epsilon-1}\frac{\lambda n}{1-\phi}} \left(\frac{L_N}{\epsilon(1 + \frac{1}{\beta})}\right)^{\lambda}\right]^{\frac{1}{1-\phi}}$$

(b) The consumption in each kind of intermediate goods, the total aggregate consumption and the per capita aggregate consumption are,

$$c_N^o = \frac{1}{\frac{1+\beta}{2}} \frac{L_N}{N_N}, \quad c_N^n = \frac{\epsilon - 1}{\epsilon} \beta \frac{1}{\frac{1+\beta}{2}} \frac{L_N}{N_N}, \quad C_N = \left(\frac{1+\beta}{2}\right)^{\frac{1}{\epsilon-1}} N_N^{\frac{1}{\epsilon-1}} L_N, \quad c_N = \left(\frac{1+\beta}{2}\right)^{\frac{1}{\epsilon-1}} N_N^{\frac{1}{\epsilon-1}}$$

(c) the nominal and real wage rate are,

$$\text{the nominal wage} \quad w_N = \left(\frac{1+\beta}{2}\right)^{\frac{1}{\epsilon-1}} N_N^{\frac{1}{\epsilon-1}}, \quad \text{the real wage} \quad \frac{w_N}{P_N} = \left(\frac{1+\beta}{2}\right)^{\frac{1}{\epsilon-1}} N_N^{\frac{1}{\epsilon-1}}$$

(d) the labor allocation is,

$$L_N^o = \frac{1}{1+\beta}L_N, \quad L_N^n(= \frac{\beta}{1+\beta}\frac{\epsilon-1}{\epsilon}L_N, \quad L_N^R = \frac{\beta}{1+\beta}\frac{1}{\epsilon}L_N$$

(e) the growth rate of innovation, wage and per capita consumption are,

$$g_{N_N} = \frac{\lambda n}{1-\phi}, \quad g_{w_N} = g_{c_N} = \left(\frac{1}{\epsilon-1}\right)\frac{\lambda n}{1-\phi}$$

Where, c_N^o is the total consumption for each old intermediate goods, C_N is total amount of aggregate consumption, c_N is per capita amount of aggregate consumption and g_{c_N} is the growth rate of per capita consumption. Price index in N is normalized to 1.

$\beta = \left(\frac{\epsilon-1}{\epsilon}\right)^{\epsilon-1} < 1.$

Similarly, we have the result for S. The derivation is in Appendix.

Result 2. *On the BGP in S,*

(a) The number of intermediate goods is, (all t is omitted)

$$N_S = N_S^o = \left[\frac{\eta}{(\epsilon-1)(\rho - g_{P_S} + \frac{\theta+\epsilon-2}{\epsilon-1}\frac{\gamma n}{1-\phi})}\left(\frac{L_S}{\epsilon}\right)^{\gamma}(\epsilon-1)\right]^{\frac{1}{1-\phi}}$$

(b) The consumption in each kind of intermediate goods, the total aggregate consumption and the per capita aggregate consumption are,

$$c_S^o = \frac{\epsilon-1}{\epsilon}\frac{L_S}{N_S}, \quad C_S = \frac{\epsilon-1}{\epsilon}N_S^{\frac{1}{\epsilon-1}}L_S, \quad c_S = \frac{\epsilon-1}{\epsilon}N_S^{\frac{1}{\epsilon-1}}$$

(c) the nominal and real wage rate are,

$$\textit{the nominal wage} \quad w_S = \frac{\epsilon-1}{\epsilon}P_S N_S^{\frac{1}{\epsilon-1}}, \quad \textit{the real wage} \quad \frac{w_S}{P_S} = \frac{\epsilon-1}{\epsilon}N_S^{\frac{1}{\epsilon-1}}$$

(d) the labor allocation is,

$$L_S^o = \frac{\epsilon - 1}{\epsilon} L_S, \quad L_S^R = \frac{1}{\epsilon} L_S$$

(e) the growth rate of innovation, wage and per capita consumption are,

$$g_{N_S} = \frac{\gamma n}{1 - \phi}, \quad g_{w_S} - g_{P_S} = g_{c_S} = \left(\frac{1}{\epsilon - 1}\right) \frac{\gamma n}{1 - \phi}$$

Where, c_S^o is the total consumption for each old intermediate goods, C_S is total amount of aggregate consumption, c_S is per capita amount of aggregate consumption and g_{c_S} is the growth rate of per capita consumption.

In autarky, there are mainly three differences between N and S. First, for usual parameters ($\theta = 1, \psi < 1, \lambda < 1, \epsilon$ not very high) and same population, two opposite forces drive the number of innovations in N and S. The difference of efficiency in doing R&D results in more innovation in N. But the competitive old sector in N absorbs some labor and makes N to have less innovation. Higher efficiency leads to the term $\frac{\lambda n}{1-\phi}$ bigger than $\frac{\gamma n}{1-\phi}$ and $\left(\frac{L_N}{\epsilon}\right)^\lambda > \left(\frac{L_S}{\epsilon}\right)^\gamma$. Labor absorbtion results in $\left(\frac{L_N}{\epsilon(1+\frac{1}{\beta})}\right) < \left(\frac{L_S}{\epsilon}\right)$. What's the outcome really depends on the tradeoff between the two forces, population, and technology externality parameters (ψ, λ, γ) and substitute parameters (θ, ϵ).

Second, N has higher welfare than S. Given the number of intermediate goods, consumption per capita and real wage in N are higher than in S. The term $\left(\frac{1+\beta}{2}\right)^{\frac{1}{\epsilon-1}} > (\beta)^{\frac{1}{\epsilon-1}} = \frac{\epsilon-1}{\epsilon}$ since $\beta < 1$. Old goods are produced competitively, thus the price relative to wage is lower and people can consume more. Another to say this is the same amount of wage can buy more old goods, or a higher real wage.

Third, the allocation of labor between production and R&D in innovation-drive sector is the same in N and S. $\frac{L^R}{L^v} = \frac{1}{\epsilon-1}$. On the BGP, labor allocation is stable, every kind of labor composes a certain ratio of population. Both production workers and R&D workers consume the same amount of goods and earn the same amount of wage. So, the ratio of R&D workers to the production workers is the mark-up (subtracted by 1) of the sector, which is same in N and S in my model. However, the ratio of R&D worker to population is smaller in N because of the existence of competitive sector.

2.2 Autarky with Offshoring

Now let's introduce offshoring into N and S economies. In this section, N offshores some parts of new goods to S. The wage rate is lower in S, so by let S to produce part of the goods, firms in N can earn more profits. But there is a cost of offshoring, so it won't be the most profitable to produce the entire good in S. Since the old sector is competitive, firms earn zero-profit and thus are indifferent between offshoring or not. To make things simple, I assume offshoring occurs only in new sector.

To isolate the effect of offshring, I assume that there is no direct trade of intermediate goods. Or equivalently, parts of new goods cannot be replaced by old goods, the offshoring sector and production sector are different in S. Workers doing offshoring are paid in N's final good. To make them indifferent between working in offshoring sector or traditional sectors in S, they are paid $w_S(t)\frac{P_N(t)}{P_S(t)}$ amount of N's final good. Other things are the same as in Section 2.1. N can produce both new goods and old goods while S only produce old goods.

Now let's state what is offshore and offshoring cost formally. My setting is very similar to Grossman and Rossi-Hansberg (2008) and Baldwin (2010). Firstly, we all consider continuous heterogenous offshoring costs. Secondly, we all assume total offshore ($\tilde{y} = 1$) is unprofitable.

Definition 3. ***Offshore and Offshoring Cost.***

A new product contains different parts $y \in [0, 1]$. Offshoring means that N produces some parts in $[0, 1]$ in S. That is N can "teach" S how to produce some parts and produce them in S. There is a cost to offshoring and it is proportional to wage in N. So to offshore part y, it costs $w_N \psi(y)$.

Increasing Offshoring Cost Assumption. *Parts $y \in [0, 1]$ ranks in offshoring cost. The bigger y, the more costly to offshore it. That is $\psi'(y) > 0$.*

Furthermore, I assume that $\psi(0) = 0$ and $\psi(1) = 1$.

Notice that the except the monotone increasing property, other assumptions on offshoring cost are just technical ones. Other combination, such as $\psi(0) = 0$ and $\psi(1) = \infty$, that offshoring costs is proportional to wage in S, will result in same qualitative results.

Since the offshoring cost is increase in parts, firms will offshore parts that are in a closed connected set, as stated formally in the following lemma.

Lemma 1. *If N offshores parts in $[y_1, y_2] \cup [y_3, y_4]$ $(0 \leq y_1 < y_2 < y_3 < y_4)$, then it offshores **all** parts in $[0, y_4]$. That is N always offshores a closed connected set of parts with 0 as its lower bound.*

Proof. By the Increasing Offshoring Cost Assumption. □

This simplifies the problem by limiting the decision variable from a set to a number.

Now let's consider firms' behavior in the new sector in N. With the option of offshoring, firms in N faces a new profit maximization problem as the following.

$$\max_{\tilde{y}(t) p_N^n(t)} \{p_N^n(t) - w_S(t) \frac{P_N(t)}{P_S(t)} \tilde{y}(t) - w_N(t)(1 - \tilde{y}(t)) - w_N(t) \int_0^{\tilde{y}(t)} \psi(y) \mathrm{d}y\} c_N^n(t) \quad (23)$$

$$s.t. \quad c_N^n(t) = (\frac{p_N^n(t)}{P_N(t)})^{-\epsilon} C_N(t) \quad (24)$$

This gives the new optimal conditions and profit.

$$\frac{w_N(t) - w_S(t)\frac{P_N(t)}{P_S(t)}}{w_N(t)} = \psi(\tilde{y}(t)), \tag{25}$$

$$p_N^n(t) = \frac{\epsilon}{\epsilon-1}[w_S(t)\frac{P_N(t)}{P_S(t)}\tilde{y}(t) - w_N(t)\tilde{y}(t) + w_N(t) + w_N(t)\int_0^{\tilde{y}(t)}\psi(y)\mathrm{d}y] = \frac{\epsilon}{\epsilon-1}\sigma(t)w_N(t) \tag{26}$$

$$\pi_N^n(t) = \frac{1}{\epsilon-1}w_N(t)\sigma(t)c_N^n(t) \tag{27}$$

where $\sigma(t) = 1 - \tilde{y}(t)\psi(\tilde{y}(t)) + \int_0^{\tilde{y}(t)}\psi(y) < 1$ since $\psi'(y) > 0$. Also, notice that $\pi_N(n,t) = \frac{1}{\epsilon-1}\left(w_N(t)\sigma(t)\right)^{1-\epsilon}C_N(t) > \frac{1}{\epsilon-1}\left(w_N(t)\right)^{1-\epsilon}C_N(t)$.

Firms producing old goods in N and S are the same as in section 2.1. That is firms in N are competitive while in S has some monopoly power.

For the main part of the paper, I assume that innovation process in N are not affected by offshoring, so they still follow (6)-(8). But the innovation process in S benefits from offshoring. The reason is as follows. As assumed before, parts of new goods cannot be replaced by old goods. So it is impossible for S to produce parts of new goods without gain some technology regarding the new goods. I describe this "technology benefit" of doing offshoring as a direct increment in S's innovation process. This increment is proportional to the increment in new goods. So the innovation process in S becomes,

$$\dot{N}_S^o(t) = \eta\left(L_S^R(t)\right)^\gamma N_S^o(t)^\phi N_N^n(t)^\alpha. \tag{28}$$

From (28), we can see that on aggregate, it seem as if S is more efficient in doing R&D after doing offshoring. If we still assume that the innovation follows a similar process as in (9), $\dot{N}_S^o(t) = \eta \left(L_S^R(t)\right)^x N_S^o(t)^\phi$, then it seems like that γ increase to x. However, this is completely due to the direct technology benefit from N.

For this section, I assume that N's innovation profits are not affected by offshoring. So the value of innovation and related non-arbitrage condition are the same as (10)(11). But the profit function changes. Combine (27)(10)(11), we get the non-arbitrage condition for N in an economy with offshoring,

$$\frac{w_N(t)}{\eta \left(L_N^R(t)\right)^{\lambda-1} N_N^n(t)^\phi} = \frac{1}{\epsilon - 1} \int_t^\infty e^{-\int_0^x r(x)dx} \sigma(t) w_N(s) c_N^n(s) ds \tag{29}$$

Combine (28)(10)(11), we get the non-arbitrage condition for S in an economy with offshoring,

$$\frac{w_S(t)}{\eta \left(L_S^R(t)\right)^{\gamma-1} N_S^o(t)^\phi N_N^n(t)^\alpha} = \frac{1}{\epsilon - 1} \int_t^\infty e^{-\int_0^x r(x)dx} w_S(s) c_S^o(s) ds \tag{30}$$

Now let's describe the equilibrium with offshoring formally. The optimal offshoring threshold connects the N and S, so though there is no trade of intermediate goods between countries, they are in an integrated world. So we define the integrated equilibrium.

Definition 4. *An autarky equilibrium with offshoring is a set of sequences of consumptions $\{c_j^v(t)\}$, sequences of prices and wage rate $\{p_j^v(t)\}$, $\{w_j(t)\}$, sequences of labor supply $\{L_N^n(t)\}$, $\{L_N^o(t)\}$,$\{L_N^R(t)\}$,$\{L_S^o(t)\}$,$\{L_S^R(t)\}$, $\{L_S^s(t)\}$(where $j = N, S$ and $v = n, o$ for N $v = o$ for S) and a sequence of offshoring parts $\{\tilde{y}(t)\}$, which is determined by the following conditions.*

(a) The represented household optimize their consumptions. So demand function follows (1), that is

$$c_j^v(t) = (\frac{p_j^v(t)}{P_j(t)})^{-\epsilon} C_j(t) = p_j^v(t)^{-\epsilon} I_j(t) P_j(t)^{\epsilon-1}.$$

$$I_j(t) = w_j(t)L_j(t)$$

(b) Firms maximize their profits.

$$p_N^n(t) = \frac{\epsilon}{\epsilon - 1}\sigma(t)w_N(t).$$

$$p_N^o(t)) = w_N(t),$$

$$P_N(t) = (N_N^n(t)p_N^n(t)^{1-\epsilon} + N_N^o(t)p_N^o(t)^{1-\epsilon})^{\frac{1}{1-\epsilon}} = 1$$

$$p_S^o(t) = \frac{\epsilon}{\epsilon - 1}w_S(t).$$

$$P_S(t) = (N_S^o(t)p_S^o(t)^{1-\epsilon})^{\frac{1}{1-\epsilon}}$$

(c) Goods market clears.

$$c_N^n(t) = \frac{L_N^n(t)}{(1 - \tilde{y}(t))N_N^n(t))} = \frac{L_S^s(t)}{\tilde{y}(t)N_N^n(t))},$$

$$c_N^o(t) = \frac{L_N^o(t)}{N_N^o(t))}.$$

$$c_S^o(t) = \frac{L_S^o(t)}{N_S^o(t))}.$$

(d) Labor market clears.

$$L_N^n(t) + L_N^o(t) + L_N^R(t) = L_N.$$

$$L_S^o(t) + L_S^R(t) + L_S^s(t) = L_S.$$

(e) Free entry of R & D sector.

$$\frac{w_N(t)}{\eta\left(L_N^R(t)\right)^{\lambda-1} N_N^n(t)^{\phi}} = \frac{1}{\epsilon - 1}\int_t^{\infty} e^{-\int_0^x r(x)dx}\sigma(t)w_N(s)c_N^n(s)ds$$

$$\frac{w_S(t)}{\eta\left(L_S^R(t)\right)^{\gamma-1} N_S^o(t)^{\phi} + \alpha\eta\left(L_N^R(t)\right)^{\lambda}\left(L_S^R(t)\right)^{-1} N_N^n(t)^{\phi}} = \frac{1}{\epsilon - 1}\int_t^{\infty} e^{-\int_0^x r(x)dx}w_S(s)c_S^o(s)ds$$

(f) Optimal offshoring threshold,

$$\frac{w_N(t) - w_S(t)\frac{P_N(t)}{P_S(t)}}{w_N(t)} = \psi(\tilde{y}(t))$$

(g) Innovation process,

$$\dot{N}_N^n(t) = \eta \left(L_N^R(t)\right)^{\lambda} N_N^n(t)^{\phi},$$

$$\dot{N}_N^o(t) = \eta \left(L_N^R(t)\right)^{\lambda} N_N^o(t)^{\phi},$$

$$\dot{N}_N(t) = \eta \left(L_N^R(t)\right)^{\lambda} N_N^n(t)^{\phi} + \eta \left(L_N^R(t)\right)^{\lambda} N_N^o(t)^{\phi}.$$

$$\dot{N}_S^o(t) = \eta \left(L_S^R(t)\right)^{\gamma} N_S^o(t)^{\phi} N_N^n(t)^{\alpha}$$

where $\sigma(t) = 1 - \tilde{y}(t)\psi(\tilde{y}(t)) + \int_0^{\tilde{y}(t)} \psi(y) < 1$

2.2.1 BGP equilibrium with offshoring

Now let's look at the innovations, consumption, wage, labor allocation and growth rate in N and S on the BGP. In a world connected by offshoring, there are several features. First, because N and S are integrated by offshoring, on the BGP, the real variables in N and S grow at the same rate and the offshoring threshold is constant.

Second, offshoring frees some labor in N from production, so the ratio of R&D workers to production workers in new sector in N increases. For S, doing offshoring moves some labor rom R&D sector to production but gain a same amount of real wage. Thus offshoring reallocates labor and will change the amount of innovations in equilibrium.

Third, the wage and consumption in each country changes upon the changes in innovation. In addition, offshoring changes the mark-up of new goods and thus the relative demands between new and old goods. In equilibrium, the consumptions and wages adjusted according to the mark-up.

Now, let's see these results in detail. First, offshoing share is constant on BGP because the new and old sector grow at the same rate. Besides, they are priced in similar way. The following lemma states this.

Lemma 2. *On the BGP, the offshoring threshold $\tilde{y}(t)$ is constant and innovations in N and S grow at the same rate. The difference between the growth rate in S and N is the growth rate of price of S's final good.*

Proof. From (13), $g_{N_N^n} = g_{N_N^o} = g_{N_N}$.
From pricing condition (b) in Definition 4, $g_{p_N^n} = \frac{1}{\epsilon-1} g_{N_N^n}$ and $g_{p_N^o} = \frac{1}{\epsilon-1} g_{N_N^o}$.
$g_{p_N^n} = g_{w_N} + g_\sigma$ and $g_{p_N^o} = g_{w_N}$. So, $g_{p_N^n} = g_{p_N^o}$.
So $g_\sigma = 0$. So $\sigma'(t) = 0$.
By the definition of $\sigma(t)$, $\sigma'(t) = [-\tilde{y}\psi'(\tilde{y}) - \psi(\tilde{y}) + \psi(\tilde{y})]\tilde{y}'(t) = -\tilde{y}\psi'(\tilde{y})\tilde{y}'(t) = 0$.
By (17), $\tilde{y} \neq 0$ generally and $\psi'(\tilde{y}) > 0$ by assumption. So $\tilde{y}'(t) = 0$, i.e. the offshoring threshold $\tilde{y}(t)$ is constant.

Then by (25), $g_{w_N} - g_{w_S} = -g_{P_S}$. i.e. The difference between the growth rate in S and N is the growth rate of price of S's final good.
From the above, we know that $g_{w_N} = \frac{1}{\epsilon-1} g_{N_N}$. Similarly, from pricing condition (b) in Definition 4,we can have $g_{w_S} = \frac{1}{\epsilon-1} g_{N_S} - g_{P_S}$
So, $g_{N_N} = g_{N_S}$. i.e. Innovations in N and S grow at the same rate. □

The main mechanism behind this seeming striking result is that offshoring connects N and S and makes them an integrated world. When there is offshoring, on the one hand, the pricing rule of new goods ((b) in Definition 4) tells us that the price relative to wage decreases and depends on share of offshoring. On the other hand, the optimal share of offshoring following (25) depends on the wage rario between N and S. On the

BGP, price and wage in N grow at the same rate. So the share of offshoring is constant and the wage ratio between N and S is constant.

With lemma 2, the structure of general equilibrium with offshoring doesn't change since $\tilde{y}$ and all related functions enter as constants. We can solve the general equilibrium in two steps. First, for given $\tilde{y}$, the system can be solved just the same as in autarky. Second, use the optimal offshoring share condition (25) to back out $\tilde{y}$. The result is stated as the following and derivation is in Appendix.

Result 3. *On BGP,*

(a) The number of intermediate goods is, (all t is omitted and remember P_N is normalized to 1.)

$$N_N = 2N_N^n = 2\left[\frac{\eta}{\rho + \frac{\theta+\epsilon-2}{\epsilon-1}\frac{\lambda n}{1-\phi}}\left(\frac{L_N}{1+\frac{1}{\beta}\sigma^{\epsilon-1}}\right)^{\lambda}(1-\frac{\frac{\epsilon-1}{\epsilon}(1-\tilde{y})}{\sigma})^{\lambda-1}\frac{1}{\epsilon}\right]^{\frac{1}{1-\phi}}$$

$$N_S = N_S^o = \left[\frac{\eta}{(\epsilon-1)(\rho - g_{P_S} + \frac{\theta+\epsilon-2}{\epsilon-1}\frac{\gamma n}{1-\phi})}\frac{\epsilon-1}{\epsilon}L_S(\frac{1}{\epsilon}L_S - L_S^s)^{\gamma-1}{N_N^n}^{\alpha}\right]^{\frac{1}{1-\phi}}$$

(b) The consumption in each kind of intermediate goods, the total aggregate consumption and the per capita aggregate consumption are,

$$c_N^o = \frac{1}{\frac{1+\beta\sigma^{1-\epsilon}}{2}}\frac{L_N}{N_N}, \quad c_N^n = \frac{\epsilon-1}{\epsilon}\beta\sigma^{-\epsilon}\frac{1}{\frac{1+\beta\sigma^{1-\epsilon}}{2}}\frac{L_N}{N_N},$$

$$C_N = \left(\frac{1+\beta\sigma^{1-\epsilon}}{2}\right)^{\frac{1}{\epsilon-1}} N_N^{\frac{1}{\epsilon-1}}L_N,, \quad c_N = \left(\frac{1+\beta\sigma^{1-\epsilon}}{2}\right)^{\frac{1}{\epsilon-1}} N_N^{\frac{1}{\epsilon-1}}$$

$$c_S^o = \frac{\epsilon-1}{\epsilon}\frac{L_S}{N_S}, \quad C_S = \frac{\epsilon-1}{\epsilon}N_S^{\frac{1}{\epsilon-1}}L_S, \quad c_S = \frac{\epsilon-1}{\epsilon}N_S^{\frac{1}{\epsilon-1}}$$

(c) the nominal and real wage rate are,

$$\textit{the nominal wage} \quad w_N = \left(\frac{1+\beta\sigma^{1-\epsilon}}{2}\right)^{\frac{1}{\epsilon-1}} N_N^{\frac{1}{\epsilon-1}},$$

$$\textit{the real wage} \quad \frac{w_N}{P_N} = \left(\frac{1+\beta\sigma^{1-\epsilon}}{2}\right)^{\frac{1}{\epsilon-1}} N_N^{\frac{1}{\epsilon-1}}$$

$$\textit{the nominal wage} \quad w_S = \frac{\epsilon-1}{\epsilon} P_S N_S^{\frac{1}{\epsilon-1}}, \quad \textit{the real wage} \quad \frac{w_S}{P_S} = \frac{\epsilon-1}{\epsilon} N_S^{\frac{1}{\epsilon-1}}$$

(d) the labor allocation is,

$$L_N^o = \frac{1}{1+\beta\sigma^{1-\epsilon}} L_N, \quad L_N^n = \frac{(1-\tilde{y})\beta}{\sigma^{\epsilon}+\beta\sigma} \frac{\epsilon-1}{\epsilon} L_N,$$

$$L_N^R = \frac{\beta\sigma - (1-\tilde{y})\frac{\epsilon-1}{\epsilon}\beta}{\sigma^{\epsilon}+\beta\sigma} L_N$$

$$L_S^o = \frac{\epsilon-1}{\epsilon} L_S, \quad L_S^R = \frac{1}{\epsilon} L_S - L_S^s, \quad L_S^s = \frac{\tilde{y}}{1-\tilde{y}} L_N^n$$

(e) the growth rate of innovation, wage and per capita consumption are,

$$g_{N_N} = g_{N_S} = \frac{\lambda n}{1-\phi}, \quad g_{w_N} = g_{c_N} = \left(\frac{1}{\epsilon-1}\right) \frac{\lambda n}{1-\phi} \quad g_{w_S} = g_{w_N} + g_{P_S}$$

(f) the offshoring share is constant and determined by,

$$\frac{w_N(t) - w_S(t)\frac{P_N(t)}{P_S(t)}}{w_N(t)} = \psi(\tilde{y}(t))$$

Where, c_N^o is the total consumption for each old intermediate goods, C_N is total amount of aggregate consumption, c_N is per capita amount of aggregate consumption and g_{c_N} is the growth rate of per capita consumption. Price index in N is normalized *to 1. c_S^o is the total consumption for each old intermediate goods, C_S is total amount of aggregate consumption, c_S is per capita amount of aggregate consumption and g_{c_S} is the growth rate of per capita consumption.*

$\beta = \left(\frac{\epsilon-1}{\epsilon}\right)^{\epsilon-1} < 1$. $\sigma = 1 - \tilde{y}\psi\tilde{y} + \int_0^{\tilde{y}} \psi(y) < 1$

3 How Offshoring Affects N And S?

Offshoring affects labor allocation, innovation, consumption and wage in both N and S. The main mechanisms are labor reallocation and changes in relative demand. Compare result 3 with result 1 and 2, we can see how offshoring plays a role.

First, we discuss the labor reallocation due to offshoring. Offshoring lowers down the mark-up of new goods and thus decreases the relative demand for old goods (relative demand effect). This leads to the shrink of the old sector and expansion of new sector in N. So less labor in the old sector while more labor in the new sector. Moreover, the ratio of R&D workers to production workers in new sector increases because some parts of new goods are done by labor of S (save-labor effect). However, we cannot tell whether there is more or less labor in the production of new sector because the two effects (relative demand effect and save-labor effect) have opposite impacts on production labor force in new sector.

For S, since the mark-up of its own monopoly sector doesn't change, the labor shore of production remains the same. However, as part of the labor force are doing offshoring, the R&D sector shrinks in S. The following proposition states the labor reallocation result.

Proposition 1. *With offshoring, in N, new sector expands and old sector shrinks. That is $L_N^n + L_N^R$ increases while L_N^o decreases. Moreover, the ratio of R&D workers to* *production workers in new sector increases.*

In S, the labor force in traditional sector remains the same while the R&D sector shrinks. That is L_S^o remains unchanged and L_S^R decreases.

Proof. For N, since $\sigma < 1$ and $\epsilon > 1$, $\sigma^{1-\epsilon} > 1$. Also $0 < \tilde{y} < 1$
Also, since $\int_0^{\tilde{y}} \psi(y) > 0$, $\psi \leq 1$, we have $\sigma = 1 - \tilde{y}\psi\tilde{y} + \int_0^{\tilde{y}} \psi(y) = 1 + \int_0^{\tilde{y}} \psi(y) - \tilde{y} + \tilde{y}(1 - \psi(\tilde{y})) > 1 - \tilde{y}$.
So, compare result 1 and 3, we have,

$$L_{N\,\text{offshore}}^o = \frac{1}{1+\beta\sigma^{1-\epsilon}} L_N < \frac{1}{1+\beta} L_N = L_{N\,\text{autarky}}^o,$$

So, $L_N^n + L_N^R = L_N - L_N^o$ increases.

$$\frac{L_{N\,\text{offshore}}^n}{L_{N\,\text{offshore}}^R} = \frac{(1-\tilde{y})\beta\frac{\epsilon-1}{\epsilon}}{\beta\sigma - (1-\tilde{y})\frac{\epsilon-1}{\epsilon}\beta} = \frac{\sigma}{1-\tilde{y}}\frac{\epsilon}{\epsilon-1} - 1 > \frac{\epsilon}{\epsilon-1} - 1 = \frac{1}{\epsilon-1}$$
$$\frac{L_{N\,\text{autarky}}^n}{L_{N\,\text{autarky}}^R} = \frac{1}{\epsilon-1}$$

So

$$\frac{L_{N\,\text{offshore}}^n}{L_{N\,\text{offshore}}^R} > \frac{L_{N\,\text{autarky}}^n}{L_{N\,\text{autarky}}^R}.$$

For S, compare result 2 and 3, we have,

$$L_{S\text{offshore}}^o = L_{S\text{autarky}}^o = \frac{\epsilon-1}{\epsilon} L_S,$$

$$L_{S\,\text{offshore}}^R = \frac{1}{\epsilon} L_S - L_S^s < \frac{1}{\epsilon} L_S = L_{S\,\text{autarky}}^R.$$

□

Second, let's see offshoring's impact on R&D sector. Two the forces mentioned above drive the R & D in N sector. The "save-labor" effect moves more workers into R&D sector. Whether this has positive or negative effect depends on the externality of more R&D workers. The ”relative demand” effect prompts the new sector and thus the amount of innovations.

For S, there are two reinforcing impact of offshoring. First is the direct technology benefit from N. Old sector in S will develop quicker than before because it absorb some parts of new products in N. Second is the labor reallocation effect. The outcome also depends the externality parameters of innovation. The following proposition states the innovation result.

Proposition 2. *Offshoring has a Save Labor Effect and a Offshore Cost Effect in N. It has a Tech Diffsution Effect and a Labor Reallocation Effect in S. The four effects with different innovation externality parameters are as in the following table.*

	$\phi < 1$ $\gamma, \lambda < 1$	$\phi < 1$ $\gamma, \lambda > 1$	$\phi > 1$ $\gamma, \lambda < 1$	$\phi > 1$ $\gamma, \lambda > 1$
Save Labor	-	+	+	-
Relative Demand	+	+	-	-
Balanced Outcome	*uncertain*	+	*uncertain*	-
Tech Diffusion	+	+	-	-
Labor Reallocation	+	-	-	+
Balanced Outcome	+	*uncertain*	-	*uncertain*

Proof. The Save Labor effect is shown in the term $1 - \frac{\frac{\epsilon-1}{\epsilon}(1-\tilde{y})}{\sigma}$ and the Relative Demand effect is shown in the term $\frac{1}{1+\frac{1}{\beta}\sigma^{\epsilon-1}}$. The Tech Diffusion effect is shown in the term ${N_N^n}^\alpha$ and the Labor Reallocation effect is shown in the term L_S^R.

Suppose, $\phi < 1$ and $\gamma, \lambda < 1$.

$\sigma > 1 - \tilde{y}$, so, $1 - \frac{\frac{\epsilon-1}{\epsilon}(1-\tilde{y})}{\sigma} > 1 - \frac{\epsilon-1}{\epsilon} = \frac{1}{\epsilon}$

$\phi < 1$ and $\lambda < 1$, so the save labor effect is negative.

$\sigma < 1$, so $\left(\frac{L_N}{1+\frac{1}{\beta}\sigma^{\epsilon-1}}\right)^\lambda > \left(\frac{L_N}{1+\frac{1}{\beta}}\right)^\lambda$.

$\phi < 1$, the relative demand effect is positive.

With $\phi < 1$, ${N_N^n}^\alpha$ has a positive effect. L_S^R is smaller with offshoring, so when $\phi < 1$ and $\gamma < 1$, L_S^R has a positive effect as well.

For other externality parameters, we can derive the results similarly. □

Third, offshoring also changes each country's welfare. Consumption and wage changes. For N, offshoring influences consumption per capita and real wage in two aspects. First, offshoring affects the number of kinds of goods. Second, offshoring lowers the mark-up of new goods and thus the price of new goods. New goods becomes cheaper relative to wage and old goods. So, the buying power of consumers in N increases. Welfare change due to number of innovations is consistent with the change in innovations. The relative price effect leads to more consumption in N. Same logic applies for wage. Wage changes according to the changes in number of innovations. More innovations makes working in both R&D and production sector more valuable, and thus increase the wage. The smaller mark-up of new goods enhances the buying power of workers in N. In other words, they have a higher real wage.

For S, people can enjoy more aggregate consumption per capita simply because the number of innovations, and thus the goods increases. Also this increases real wage level in S. The technology benefit improves S's welfare. The actual outcome depends on innovation externality parameters since the number of innovations depends on them. The following proposition states the result for consumption and wage.

Proposition 3. *Offshoring lowers down the price of new goods in N, and thus increases the aggregate consumption per capita and real wage. However, consumption and wage also depends on the number of innovations. So, the balanced-out result depends on offshoring's impact on innovations, which is stated in the above proposition. For S, offshoring influences the aggregate consumption per capita and real wage simply because* *it influence the number of innovations.*

Proof. For N, the relative price effect is shown in the term $\sigma^{1-\epsilon}$, the innovation effect is shown in the term N_N.

For S, the innovation effect is shown in the term N_S.

For N, from result 1 and 3, we know that

$$c_{N\text{offhsoring}} = \left(\frac{1+\beta\sigma^{1-\epsilon}}{2}\right)^{\frac{1}{\epsilon-1}} N_N^{\frac{1}{\epsilon-1}} > \left(\frac{1+\beta}{2}\right)^{\frac{1}{\epsilon-1}} N_N^{\frac{1}{\epsilon-1}} = c_{N\text{autarky}}$$

$$w_{N\text{offhsoring}} = \left(\frac{1+\beta\sigma^{1-\epsilon}}{2}\right)^{\frac{1}{\epsilon-1}} N_N^{\frac{1}{\epsilon-1}} > \left(\frac{1+\beta}{2}\right)^{\frac{1}{\epsilon-1}} N_N^{\frac{1}{\epsilon-1}} = w_{N\text{autarky}}$$

if N_N remains the same. So, offshoring increases the aggregate consumption per capita and wage in N given N_N.

$$\frac{\partial}{\partial N_N} w_N = \left(\frac{1+\beta\sigma^{1-\epsilon}}{2}\right)^{\frac{1}{\epsilon-1}} \frac{1}{\epsilon-1} N_N^{\frac{-\epsilon}{\epsilon-1}} > 0$$

$$\frac{\partial}{\partial N_N} c_{N\text{offhsoring}} = \left(\frac{1+\beta\sigma^{1-\epsilon}}{2}\right)^{\frac{1}{\epsilon-1}} \frac{1}{\epsilon-1} N_N^{\frac{-\epsilon}{\epsilon-1}} > 0$$

So, offshoring influences the aggregate consumption per capita and wage as the same way of it influences innovations.

For S, from result 2 and 3, we know that

$$c_{S\text{offhsoring}} = \frac{\epsilon-1}{\epsilon} N_S^{\frac{1}{\epsilon-1}}$$

$$c_{S\text{autarky}} = \frac{\epsilon-1}{\epsilon} N_S^{\frac{1}{\epsilon-1}}$$

$$w_{S\text{offhsoring}} = \frac{\epsilon-1}{\epsilon} P_S N_S^{\frac{1}{\epsilon-1}}$$

$$w_{S\text{autarky}} = \frac{\epsilon-1}{\epsilon} P_S N_S^{\frac{1}{\epsilon-1}}$$

So only the number of innovations will changes consumption per capita and wage in S.

From proposition 2, we know that the change in the number of innovations depends on externality parameters.

□

Finally, since we are in a semi-growth model, growth rate doesn't depend on labor allocations. So offshoring will not affect growth by reallocate labor. However, since the innovation process in S improves, S benefit from offshoring and has a higher growth rate.

4 What'S The Consequence Of Changes In Offshoring Cost?

Modern transmission technology develops rapidly and facilitates offshoring. At first glance, lower offshoring cost makes it easy to produce in S and the offshoring share should increase. However, in a general equilibrium framework, it is not necessarily true. Two important forces push the offshoring share in opposite direction. First, in a general equilibrium framework, more offshoring in will increase S's wage while decrease N's with usual innovation externality parameters ($\phi < 1, \gamma, \lambda < 1$). Second, offshoring cost affects the mark-up of new goods, and thus influences relative demand and buying power in N. For different types of offshoring cost function, the changes in mark-up differs. In some cases, a lower offshoring cost will decrease the mark-up and thus lead to more relative demand for new goods. This will increase the demand for offshoring workers in S and thus their wage. It will increase N's workers's wage as well. Which country's wage increases more depends on the parameters. Now, let's see the two forces in detail.

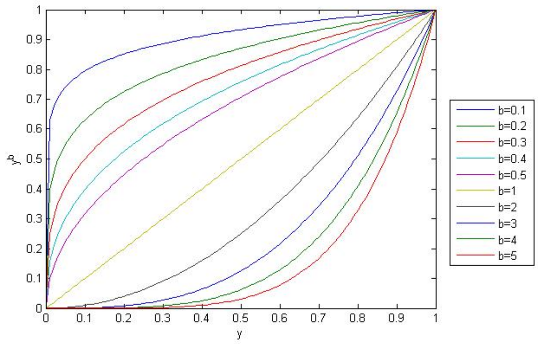

Figure 1: Offshoring cost with different b

Before beginning the analysis, let me describe how to measure the changes in offshoring cost. In this section,

$$\psi(y) = y^b$$

where b measures the level of offshoring cost.

As b increases, the cost of offshoring $\int_0^{\tilde{y}} y^b dy$ decreases, since $\frac{\partial}{\partial b}\int_0^{\tilde{y}} y^b dy = \int_0^{\tilde{y}} lnyy^b dy < 0$ as $y < 1$. Or as we can see from Figure 1, the higher the b, the smaller the area below the curve.

4.1 How offshoring share influences wage gap between N and S?

Suppose now the offshoring cost lowers down, that is b increase, is it more profitable to increase offshoring share? Firms will have desire to offshore more. By the optimal offshoring share condition (25), if the wage gap remains the same, $\tilde{y}$ will increase. However, with more offshoring, the wage gap will decrease with same innovation externality, which push back the offshoring share. In this sector, we fix σ, fix the changes in relative market size. The following proposition states this.

Proposition 4. *For given σ, When $\phi < 1, \gamma, \lambda < 1$ or ,$\phi > 1, \gamma, \lambda > 1$, $\frac{w_S(t)\frac{P_N(t)}{P_S(t)}}{w_N(t)}$ increases as $\tilde{y}$ increases. So the incentive to increase offshoring share is alleviated.*

When $\phi < 1, \gamma, \lambda > 1$ or ,$\phi > 1, \gamma, \lambda < 1$, $\frac{w_S(t)\frac{P_N(t)}{P_S(t)}}{w_N(t)}$ decreases as $\tilde{y}$ decreases. So the incentive to increase offshoring share is reenforced.

Proof. Suppose $\phi < 1, \gamma, \lambda < 1$.
From result 3, we know that as $\tilde{y}$ increases, N_N and L_N^n decreases, L_S^s increases. So N_S increases. (We assume $\alpha < 1 - \phi$, which means S cannot benefit from N's innovations more than N itself does.)
So, $w_S(t)$ increases while $w_N(t)$ decreases. So, $\frac{w_S(t)\frac{P_N(t)}{P_S(t)}}{w_N(t)}$ increases.

For other externality parameters, we can derive the results similarly. □

The main mechanism behind this proposition is the followings. Consider the case $\phi < 1, \gamma, \lambda < 1$. An increase in offshoring share will lower down the demand for workers in N and thus the wage. Though it saves labor for R&D, as researchers have negative externality, the decrease in wage will not be compensated. For S, the reason is the same. With more offshoring, the damand of labor is higher and thus the wage increases.

4.2 Relative Demand Effect

In the general equilibrium model, there is another more subtle but important channel. The changes in offshoring costs will changes the mark-up of new goods and thus the relative demand for new goods. In most part of this section, I fix y.

First, let's see how changes in costs affect the mark-up σ.

$$\frac{\partial}{\partial b}\sigma = -\frac{y^{b+1}}{(b+1)^2}(1 + b(b+1)lny)$$

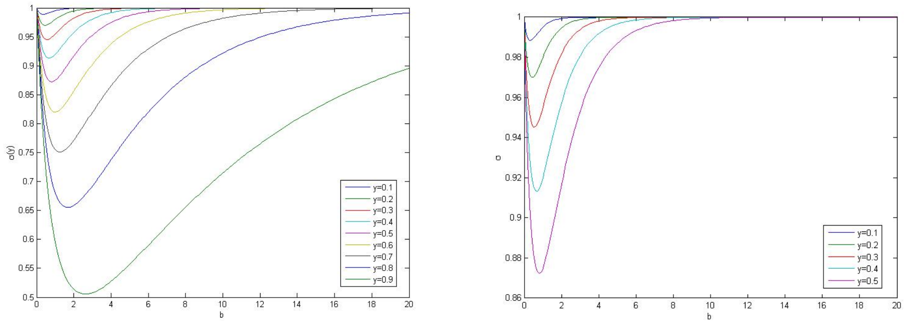

Figure 2: How mark-up (σ) changes with cost (b). The right figure the the zoom-up of the top area of the left one.

The sign depends on b and y. As shown in Figure 2, 1) σ decreases first and then increases as cost decreases, 2) For large offshoring share y, σ decreases more with the same amount of decrease of cost. 3) For large offshoring share y, σ won't increase until the cost decreases to a very low level. These features are intuitive. The price should reflect the cost, so as offshoring cost decreases, things becomes cheaper. However, the wage-gap benefit, which is measured in $-y\psi(y)$, becomes smaller, so the mark-up increases. As in the way I set up the measurement of cost, when cost is very low, the later effect dominates and thus the mark-up increases. For larger y, the direct cost effect is more important and the mark-up will decrease with a bigger range.

Second, I discuss how innovation action, welfare and wage-gap reacts to the changes in σ. Depend on different innovation externality parameters, results are different. Generally, N and S changes in the same direction except the number of workers in R&D sector. But S has more cases of welfare improvement while in most cases, the result in N is uncertain. For the wage-gap, the result is ambiguous because there are forces in same direction driving both countries' wage. The following proposition states these results.

Proposition 5. *Suppose σ increases, labor allocation, innovation, consumption and wage changes have the following properties,*

i *L_N^n and L_S^s move in the same direction and L_N^R and L_S^R move in the same direction. But production and R&D workforce move in opposite directions. That is there is co-movement between same sector across countries but the two sector move oppositely.*

ii *N_N and N_S, c_N and c_S , w_N and w_S move in the same way.*

iii *Movement of the wage-gap is uncertain.*

Proof. Suppose σ increases, it affects the variables on BGP in the following way. The term in blanket shows why the effect is negative or positive. The last line of each variable shows the total effect of σ. ”uc” represents ”uncertain”.

	$\phi < 1$ $\gamma, \lambda < 1$	$\phi < 1$ $\gamma, \lambda > 1$	$\phi > 1$ $\gamma, \lambda < 1$	$\phi > 1$ $\gamma, \lambda > 1$
L_N^n ($\frac{1}{\sigma^{\epsilon}+\beta\sigma}$)	-	-	-	-
L_N^R ($\frac{1}{\sigma^{\epsilon-1}+\beta}, -\frac{1}{\sigma^{\epsilon}+\beta\sigma}$)	+	+	+	+
L_S^s (L_N^n)	-	-	-	-
L_S^R (-L_S^s)	+	+	+	+
N_N				
Relative Market (($\frac{1}{1+\frac{1}{\beta}\sigma^{\epsilon-1}})^{\frac{1}{1-\phi}}$)	-	-	+	+
R&D workers				
($(1-\frac{\frac{\epsilon-1}{\epsilon}(1-\tilde{y})}{\sigma})^{\frac{\lambda-1}{1-\phi}}$)	-	+	+	-
	-	**uc**	**+**	**uc**
N_S				
Tech Diffusion ($N_N^{\frac{1}{1-\phi}}$)	-	uc	+	uc
R&D workers				
((($\frac{1}{\epsilon}L_S - L_S^s)^{\gamma-1})^{\frac{1}{1-\phi}}$)	-	+	+	-
	-	**uc**	**+**	**uc**
c_N				
Relative Market ($1+\beta\sigma^{1-\epsilon}$)	-	-	-	-
Innovations (N_N)	-	uc	+	uc
	-	**uc**	**uc**	**uc**
c_S				
Innovations (N_S)	**-**	**uc**	**+**	**uc**
w_N				
Relative Market $1+\beta\sigma^{1-\epsilon}$)	-	-	-	-
Innovations (N_N)	-	uc	+	uc
	-	**uc**	**uc**	**uc**
w_S				
Innovations (N_S)	**-**	**uc**	**+**	**uc**
wage gap ($\frac{w_S}{w_N}$)				
R&D workers in S ($\frac{1}{\epsilon}L_S - L_S^s)^{\frac{\gamma-1}{1-\phi}}$)	-	+	+	-
Relative Market to wage				
(($1+\beta\sigma^{1-\epsilon})^{\frac{1}{1-\phi}}$)	+	+	+	+
Relative Market to innovation				
(($1+\beta\sigma^{\epsilon-1})^{\frac{1}{1-\phi}}$)	-	-	-	-
R&D workers ($(1-\frac{\frac{\epsilon-1}{\epsilon}(1-\tilde{y})}{\sigma})^{\frac{\lambda-1}{1-\phi}}$)	+	-	-	+
	uc	**uc**	**uc**	**uc**

□

Relative demand is the reason behind. Consider the case $\phi < 1, \gamma, \lambda < 1$. The increase in mark-up will make the new sector less attractive and few workers in it (including production workers from N and S and R&D workers in N). Then the innovations in N decreases because 1) few input 2) less demand. As a result, the per capita consumption and wage in N decreases. For S, the R&D sector expands, but because of the negative externality, less innovations on average is produced. So the innovations decreases due to 1) technology benefit from N, 2) decrease returns to scale in R&D sector. And the consumption per capita and wage decreases. Since the wage changes in the same direction, how the wage-gap changes is ambiguous.

I simulate the changes in labor allocation, consumption, wage and wage gap according to different externality parameter and population at different level of offshoring. Parameters are set in the following way. $\eta = 1$, $\rho = 0.95$, $\theta = 0.8$, $\epsilon = 2.5$. Innovation externality parameters and population scale changes in different cases. Parameter Selection and innovations, consumptions and wage gap response in different scenarios are shown in Appendix. In Case 1-5, I study the how existing innovation externality affects results. In case 6 and 7, I study whether relative innovation efficiency and population scale among N and S affects results. In case 3, 5, 8, 10, I study results of different combinations of innovation externalities.

For innovation, wage and consumption, (from result 3, we know that wage and consumption move in the same way. To save places, I omit the graphs for consumption), we have the following results. 1) When $\phi < 1$, they increase first and then decrease. As ϕ approaches 1, the reaction becomes smaller. When $\phi > 1$, they decrease first and then increase. 2) When offshoring share is low, the reaction to cost changes is very small. As offshoring share becomes large, reaction enlarges. Consesquently, when $\phi < 1$, the more offshored, the higher welfare is. This is true for both countries in most cases (expect case 9). When $\phi > 1$, the more offshored, the lower welfare is. 3) When the difference in innovation efficiency between N and S is small (compare case 3 and case 6, and case 8 and case 9), that is S is just a little less efficient than N, S benefits less from offshoring at high level of offshoring share but more at low level. In case 9, offshoring cost reduction even have negative impacts on S. 4) Population scale doesn't affect the economy much (compare case 3 and case 7). The reaction pattern is the same. But a higher population reduces welfare. 5) When ϕ is the same, different λ and γ doesn't change the patterns at low level of offshoring but leads to opposite patterns in S at high level. (compare 3, 5 and 8, 10)

The intuition of these results is the following. As argued before, relative demand and labor movement between new and old sector lead to the result. When $\phi < 1$, as cost decreases, the mark-up σ decreases first and then increases. So the new sector expands first and then shrinks. As the externality is small, the welfare comoves with the new sector. When the researcher externality (λ and γ) is big, bigger than 1, one force in S changes. The R&D sector becomes increase return to scale. So as the R&D sector shrinks in S, the welfare decreases. So a decrease in mark-up decreases welfare in S. When $\phi > 1$, by just the same logic, we have the opposite result.

Explanation for other results is as follows. 1) As ϕ approaches 1, the existing innovations becomes neutral to the growth of future innovations, so the reaction becomes smaller. 2) The more offshored, the larger is the absolute change in demand. So the bigger impacts on welfare. 3) When the difference in innovation efficiency between N and S is small, S benefits less from direct tech diffusion. So the welfare in S increase less. When the research externality is large (case 9), the loss from own R&D overweighs the benefit from tech diffusion. So welfare decreases as the mark-up lowers down. 4) Population scale doesn't affect the economy much is a nature outcome of semi-growth model. A higher population reduces per capita consumption and wage is also because there is no scale effect in innovations. 5) Different λ and γ might change the patterns in S at high level because the change in own R&D sector is big (and can have chance to overweigh the tech diffusion effect) with big offshore share.

For wage gap, we can see the general trend depends on externalities. In the graphs, wage gap is measured as $1-\frac{w_S}{w_N}$. When existing externality and researcher externality is the same ($\phi < 1$, $\lambda, \gamma < 1$ or $\phi > 1$, $\lambda, \gamma > 1$), wage gap decreases and then increases. That is S benefit more than N first and then less than N. When existing externality and researcher externality is opposite, we gain opposite results. As $\phi 1$ increases, the wage gap becomes large. There are even some cases, S has a larger wage than N. When $\phi 1$ is large, the technology diffusion benefit is large. Same as of the welfare implications, when the difference in innovation efficiency between N and S is small, S benefits less from offshoring at high level of offshoring share but more at low level.

4.3 conclusion

Combining the two parts of analysis, we can see in general, the influence of reduction in offshoring costs is complex. However, we can still drive some conclusions. First, with negative existing externality ($\phi < 1$) it is more beneficial to reduce offshoring cost when it is very high (b is small). When cost is high, the wage gap effect is relatively more important and a reduction in costs will decrease the mark-up, leading to more demand. When innovation action is more difficult with a large amount of existing ones, demand is very important incentives for innovations. Second, how wage gap changes is uncertain because N and S changes in the same direction in reaction to changes in mark-up. This is because the mark-up affects the relative demand between old and new goods in N. A decrease in mark-up leads to the expansion of new sector in N, and demands more labor from both N and S in the new sector. This result is very different from how movement in offshoring share affects wage-gap. In the latter, the demand for labor moves in opposite direction.

5 Discussion

In this section, I discuss the following points. First, what is the difference between trade of goods and offshoring. Second, what if N is on BGP while S is not? Finally, some comments on endogenous growth model.

5.1 Trade of goods Vs Offshoring

In this subsection, I discuss what's the different impact of trade and offshoring. The model is in the same spirit of Grossman and Helpman (1991). I modified it to be close to my model so that impacts of trade and offshoring can be compared. Both N and S can consume new goods and old goods. I consider the specialization equilibrium where N produce the new goods with monopoly power and S produce the old goods. Since the world is integrated now, technology of old goods are prevailing. So the old sector is competitive. Also, the innovation in new sector has effects on the old sector. This can be interpreted as imitation or technology diffusion.

Similarly as in section 2, we can define a specialization equilibrium with free trade.

Definition 5. *A specialization equilibrium with free trade is a set of sequences of consumptions $\{c_j(v,t)\}$, sequences of prices and wage rate $\{p(v,t)\}$, $\{w_j(t)\}$, sequences of labor supply $\{L_N^n(t)\}$, $\{L_S^o(t)\}$,$\{L_N^R(t)\}$,$\{L_S^R(t)\}$ and the sequences of number of new and old goods $\{N_N^v(t)\}$ (where $j = N, S$ and $v = n, o$), which is determined by the following conditions.*

(a) *The represented household optimize their consumptions. So demand function follows (1), that is*

$$c_j^v(t) = (\frac{p_j^v(t)}{P_j(t)})^{-\epsilon} C_j(t) = p_j^v(t)^{-\epsilon} I_j(t) P_j(t)^{\epsilon-1}.$$

$$I_j(t) = w_j(t) L_j(t)$$

(b) *Firms in new sector maximize their profits while firms in old sector are competitive.*

$$p^n(t) = \frac{\epsilon}{\epsilon - 1} w_N(t).$$

$$p^o(t)) = w_N(t),$$

$$P_N(t) = (N_N^n(t) p_N^n(t)^{1-\epsilon} + N_N^o(t) p_N^o(t)^{1-\epsilon})^{\frac{1}{1-\epsilon}} = 1$$

$$P_S(t) = (N_S^o(t) p_S^o(t)^{1-\epsilon})^{\frac{1}{1-\epsilon}}$$

(c) *Goods market clears.*

$$c_N(n,t) + c_S(n,t) = \frac{L_N^n(t)}{N^n(t))},$$

$$c_N(o,t) + c_S(o,t) = \frac{L_S^o(t)}{N^o(t))}.$$

(d) *Labor market clears.*

$$L_N^n(t) + L_N^R(t) = L_N.$$

$$L_S^o(t) + L_S^R(t) = L_S.$$

(e) *Free entry of R & D sector.*

$$\frac{w_N(t)}{\eta \left(L_N^R(t)\right)^{\lambda-1} N_N^n(t)^{\phi}} = \frac{1}{\epsilon - 1} \int_t^{\infty} e^{-\int_0^x r(x)dx-} w_N(s) c_N^n(s) ds$$

(f) Innovation process follow

$$\dot{N}_N^n(t) = \eta \left(L_N^R(t)\right)^\lambda N_N^n(t)^\phi,$$

$$\dot{N}_N^o(t) = \eta \left(L_N^R(t)\right)^\lambda N_N^o(t)^\phi,$$

$$\dot{N}_N(t) = \eta \left(L_N^R(t)\right)^\lambda N_N^n(t)^\phi + \eta \left(L_N^R(t)\right)^\lambda N_N^o(t)^\phi.$$

$$\dot{N}_S^o(t) = \eta \left(L_S^R(t)\right)^\gamma N_S^o(t)^\phi N_N^n(t)^\theta.$$

We can see there are several similar impacts of trade and offshoring. But the mechanisms are different. Trade connects the world by rearrangement between goods production. By making foreign goods available, countries specialize in different sectors. Offshoring connects the world by rearrangement between labors. By using labor from S, N saves labor in production. By doing offshoring, labor productivity becomes more efficient in S. The long-run offshoring threshold moves to the most efficient allocation among production and R&D in both countries. So, for N, both trade and offshoring have two opposite effects. But the effects are different. As for trade, the positive effect comes from specialization. Specialization in new goods improves production efficiency. This is shown by the term $\frac{1+\beta}{\beta}$, which is bigger than 1. So trade enhances growth. The negative effect is resulted from imitation of S. Imitation from S makes R&D in N less profitable and thus less incentive to invest in R&D. This is represented by the term $\frac{\epsilon-1}{\alpha} = (\epsilon - 1)\frac{\iota L_S^R}{\eta L_N^R}$, which is bigger than 0. So trade slows down growth. Moreover, the bigger the imitation, (the bigger ι), the more growth is influences. The balance between the two effects is affected by all the parameters. Some basic results can be talked. As the desire for consumption smooth increases (the bigger θ), the positive role from specialization are more important. For large economy (with a big population) and small substitution desire (small θ), the negative imitation effect is severe.

As for offshoring, there is a positive "save labor" effect. Since offshoring "borrows" labor from S to produce new goods, N can have more labor available in R&D and thus has a higher growth rate. This is shown by the term $1 - \tilde{y}$, which is smaller than 1. The negative effect comes from the offshoring cost, which consumes some profits. As R&D is financed by monopoly rents, the offshoring cost decreases R&D incentives. This is shown by the term σ, which is less than 1. The balance between the two effects depends on the offshoring cost function and substitution between variaties.

For S, trade has two effects. First is the positive tech diffusion effect. By the possibility of enjoying new goods now, S can imitate from new goods and grows at the same rate as N. This is shown by the change from ι to η (it is reasonable to assume that $\iota < \eta$). Second, however, the imitation also has a negative effect for S. This is because imitation decreases the growth rate of N, which is the the integrated world's rate as well. This is shown by the term $\frac{\epsilon-1}{\alpha}$.

5.2 Transition to BGP

Now, I consider a more realistic situation where N is on the BGP while S isn't. If S and N don't grow at the same rate, the offshoring share is not constant. Then, as we can see from propostion 4, labor doing offshoring in S is not constant and not even grows at constant rate. So S cannot be on BGP.

Now, let's consider a simple case where ι' is exogenously given. To have different growth rates in N and S, we need change the set-up a little bit. Now, numbers of old goods and new goods in N grow at different rates. This is possible because of the offshoring. When there is no offshoring, the innovation rate equals across sectors.

So, in this subsection, the innovation processes in N follows,

$$\dot{N}_N^n(t) = \eta L_N^R(t) N_N^n(t) \tag{31}$$

$$\dot{N}_N^o(t) = \eta' L_N^R(t) N_N^o(t) \qquad (\eta' < \eta). \tag{32}$$

and the innovation processes in S follows,

$$\dot{N}_S^o(t) = \iota L_S^R(t) N_S^o(t). \tag{33}$$

The equilibrium doesn't change, still as in Definition 5.

Proposition 6. *When N is on the BGP and S isn't,* $\{L_N^n(t)\}$, $\{L_N^R(t)\}$, $\{L_S^o(t)\}$ *are constant,* $\{L_S^o(t)\}$,$\{L_S^R(t)\}$, $\{L_S^s(t)\}$ *are not.*

When $\eta' < \eta$,

$$L_N^o = \frac{\rho(\epsilon-1) + \eta(\theta+\epsilon-2)L_N}{\eta\beta(t)\sigma(t)^{1-\epsilon} + (1+\beta(t)(1-\tilde{y}(t))\sigma(t)^{-\epsilon})\eta(\theta+\epsilon-2)}$$

$$L_N^n = \beta(t)(1-\tilde{y}(t))\sigma(t)^{-\epsilon}\frac{\rho(\epsilon-1) + \eta(\theta+\epsilon-2)L_N}{\eta\beta(t)\sigma(t)^{1-\epsilon} + (1+\beta(t)(1-\tilde{y}(t))\sigma(t)^{-\epsilon})\eta(\theta+\epsilon-2)}$$

$$L_N^R = \frac{\eta\beta(t)\sigma(t)^{1-\epsilon}L_N - (1+\beta(t)(1-\tilde{y}(t))\sigma(t)^{-\epsilon})\rho(\epsilon-1)}{\eta\beta(t)\sigma(t)^{1-\epsilon} + (1+\beta(t)(1-\tilde{y}(t))\sigma(t)^{-\epsilon})\eta(\theta+\epsilon-2)}$$

$$L_S^s(t) = \frac{\tilde{y}(t)}{1-\tilde{y}(t)}L_N^n$$

$$L_S^o(t) = \frac{\rho(\epsilon-1) + \iota(\theta+\epsilon-2)(L_S - L_S^s(t))}{\iota + \iota(\theta+\epsilon-2)}$$

$$L_S^R(t) = \frac{\iota(L_S - L_S^s(t)) - \rho(\epsilon-1)}{\iota + \iota(\theta+\epsilon-2)}$$

$$g_{N_N^n} = \eta L_N^R \qquad g_{N_N^o} = \eta' L_N^R \qquad g_{w_N} = g_{C_N} = \frac{\eta L_N^R}{\epsilon-1}$$

$$g_{N_S} = \iota L_S^R(t) \qquad g_{w_S} = g_{C_S} = \frac{\iota L_S^R(t)}{\epsilon - 1} \qquad \textit{is increasing with time.}$$

where $\sigma(t) = 1 - \tilde{y}\psi(\tilde{y}) + \int_0^{\tilde{y})} \psi(y)$.

$\tilde{y}$ *is determined by* $\frac{w_N(0) - w_S(0)}{w_N(0)} = \psi(\tilde{y}(t))$.

$\beta(t) = \left(\frac{\epsilon-1}{\epsilon}\right)^{\epsilon} \frac{N^n(t)}{N^o(t)}$. *is similar as in propostion 1.*

Note that when N is on the BGP, $\beta(t)\sigma(t)^{1-\epsilon}$ *and* $\beta(t)(1 - \tilde{y}(t))\sigma(t)^{-\epsilon}$ *are constant.*

Proof. The main methodology to solve the equilibrium is that, given $\tilde{y}(t)$, we can solve the equilibrium in N as in autarky. And then, we get the labor in offshoring sector in S, so we can solve the equilibrium in S as in autarky as well. Finally, we use the optimal offshoring condition (17) to back out $\tilde{y}(t)$.

I prove the proposition by five steps.

(i) $g_\sigma = g_{1-\tilde{y}} + g_{L_N^o} - g_{L_N^n} = \frac{1}{\epsilon-1}(g_{N_N^n} - g_{N_N^o}) = \frac{1}{\epsilon-1}(\eta - \eta')L_N^R$.

From the price index in N (22), we have $g_{p^n} - g_{p^o} = \frac{1}{\epsilon-1}(g_{N_N^n} - g_{N_N^o})$.

From pricing rule of goods in N (20)(21), we have $g_\sigma = g_{w_N} = g_{p^n} - g_{p^o} = \frac{1}{\epsilon-1}(g_{N_N^n} - g_{N_N^o})$.

From demand function (1), we have $g_{C^o} - g_{C^n} = \epsilon g_\sigma$.

From goods market clear conditions, we have $g_{C^o} - g_{C^n} = (\epsilon - 1)g_\sigma + g_{1-\tilde{y}} + g_{L_N^o} - g_{L_N^n}$.

So $g_\sigma = g_{1-\tilde{y}} + g_{L_N^o} - g_{L_N^n}$. (i)

By definition of innovation processes in N (28)(29), $\frac{1}{1-\epsilon}(g_{N_N^n} - g_{N_N^o}) = \frac{1}{1-\epsilon}(\eta' - \eta)L_N^R$.

So we have $g_\sigma = g_{1-\tilde{y}} + g_{L_N^o} - g_{L_N^n} = \frac{1}{\epsilon-1}(g_{N_N^n} - g_{N_N^o}) = \frac{1}{\epsilon-1}(\eta - \eta')L_N^R$.

(ii) $g_{L_N^o} = g_{L_N^n} = g_{L_N^R} = 0$

On BGP, by definition, $g_{L_N^R} = 0$. So, by labor market clear condition in N, $g_{L_N^o} + g_{L_N^n} = 0$. (ii)

By the value function of innovation in N, we have $1 = \frac{\eta\sigma(t)L_N^n(t)}{(\epsilon-1)(1-\tilde{y}(t))(r+g_{N_N^n}-g_{w_N}-g_{L_N^n})}$.

So we have $g_\sigma = g_{1-\tilde{y}} - g_{L_N^n}$. (iii)

Combining (i)-(iii), we have $g_{L_N^o} = g_{L_N^n} = 0$.

So, $g_{L_N^o} = g_{L_N^n} = g_{L_N^R} = 0$.

So $\{L_N^n(t)\}$, $\{L_N^R(t)\}$, $\{L_S^o(t)\}$ are constant.

By solving the model in a standard way, we can get $\{L_N^n(t)\}$, $\{L_N^R(t)\}$, $\{L_S^o(t)\}$ and the growth rates in the proposition.

(iii) $g_\sigma = g_{1-\tilde{y}} = \frac{1}{\epsilon-1}(g_{N_N^n} - g_{N_N^o}) = \frac{1}{\epsilon-1}(\eta - \eta')L_N^R$.

This is directly from (i) and (ii).

So, $\beta(t)\sigma(t)^{1-\epsilon}$ and $\beta(t)(1-\tilde{y}(t))\sigma(t)^{-\epsilon}$ are constant.

(iv) $L_S^s(t)$ is decreasing with t.

$\eta' < \eta$, so $g_\sigma > 0$.

By definition of $\sigma(t)$, this means $\frac{\tilde{y}(t)}{dt} < 0$.

By goods market clearing condition (23), $L_S^s(t) = \frac{\tilde{y}(t)}{1-\tilde{y}(t)}L_N^n$.

$\frac{\tilde{y}(t)}{1-\tilde{y}(t)}$ is increasing in $\tilde{y}$, so, $L_S^s(t)$ is decreasing with t.

(v) $g_{N_S} = \iota L_S^R(t)$ is increasing with t.

By solving the model in a standard way, we can get $\{L_S^R(t)\}$, $\{L_s^o(t)\}$ and the growth rates in the proposition. Notice, the economic environment doesn't change in S by doing offshoring, just the total labor in R&D and old sector decreases by $L_S^s(t)$.

$L_S^R(t)$ is decreasing in $L_S^s(t)$, so combining (iv), $g_{N_S} = \iota L_S^R(t)$ is increasing with t.

□

Corollary 1. *When $\eta' < \eta$, optimal offshoring share $\tilde{y}(t)$ is decreasing with time, g_S is increasing and $g_S < g_N$.*
When $\eta' > \eta$, optimal offshoring share $\tilde{y}(t)$ is increasing with time, g_S is decreasing and $g_S > g_N$.

So offshoring makes it easier for S to grow (either with increasing growth rate or higher growth rate), which is the phenomenon observed nowadays. Also, since there is no change is offshoring cost and spillover effects, the offshoring share decreases (increases) as the wage gap narrows (enlarges). This is intuitive because the incentive to offshore parts in S is entirely driven by the wage gap between N and S. The bigger the wage gap, the larger the offshoring share. However, the corollary suggests that this S grows with increasing growth rate only when it grows less rapidly than N. If it grows more rapidly, the growth rate will decrease. This is because eventually, N and S will become an integrated world, connected by offshoring, and grow at the same rate.

5.3 Endogenous Growth Model

The major part of the paper is build in a semi-growth model. Will the conclusions still be true in a endogenous growth model? It is easy to set up a similar endogenous growth model. And intuitively, the labor movements and relative price change will still exist. The mechanism doesn't changes. However, as in endogenous growth model, innovation externality is assumed to be 1, results will be more certain. Also, as there are "scale-effect", labor movement between production and R&D sector will be more meaningful.

N gains from more researchers while S will loss from this. The growth rate will be affected by innovation as well.

6 Conclusion

The development of modern transmission and information technology changed the nature of trade a lot. Instead of direct trade of goods, the corporation between countries are in a form of global supply chain (or task trade as another terminology). The main difference of GSC and trade lies in the distinction between a whole process of a product and various products. Offshoring is a new and old topic. Like in Feenstra and Hanson (1996), they identified the impacts of outsourcing. There are also many researches of vertical integration in a international view. However, recent researches focus more on the cost of offshoring and decision of firms. Costs are heterogenous and play an important role. Also, how much to offshore is a optimal decision of firms.

This paper followed the new trend and studied the impacts of offshoring in a semi-endogenous growth model. This brings out an important role of offshoring: to reallocate labor between R&D and production. When externality of existing innovation and researchers are not very high, offshoring benefits both countries. It exploits each country's comparative advantage. DCs have comparative advantage in innovation while LDCs have comparative advantage in production. Also, offshoring lowers down the cost of new goods and thus the price. This will improve welfare due to demand changes. People will demand more new goods and the new sector is promoted.

The paper is only a first attempt into the question of innovation and offshoring. The model is still at its first stage. There are three important points that I hope to improve in future study. First, the technology connection between N and S should be characterized more clearly and comprehensively. How offshoring brings N's technology to S? will offshoring in S harms N's innovation rents? For example, S might imitate the whole good from parts. How much technology can S learn from those offshored parts? Are the technology regarding those parts appropriate for S? All these questions are meaningful and better to be embedded.

Second, it is interesting to enrich the model by including multiple goods and various skill level of workers. In that case, the production functions will be heterogenous. There will be richer movements in labor allocation. For example, it is possible that offshoring doesn't boost innovation a lot. Instead, ”saved” labor moves to other production sectors. This is realistic because usually innovation requires higher skill level.

Third, the whole dynamic process in response to offshoring cost changes needs more detailed research. The paper studied only the difference in variables on the BGP. The study on transition between states will provides insights and suggestions as well. Also, it is better to study the exact general equilibrium outcome as well as analyzing different channels.

I hope this first attempt provides the framework and some insights on how offshoring affects innovation and welfare. In this paper, I found that it is more beneficial to lower down cost at high existing level if the marginal cost increases as average cost decreases. This suggests that policy promoting offshore in difficult industry is more helpful than that in industries with prevailing offshoring. One main result of my paper is that innovation externality influence welfare a lot. This implies that we should not take a one-cut policy on offshoring. It should be based on the nature of the industry and products. Moreover, as different forces drive the outcome, we cannot come to a unique conclusion. So it will be very interesting to identify different impacts and calculate which are more important that others. Finally, in modern society, IPR and management are two important elements affects innovation and offshoring. Future research can consider to include them as well.

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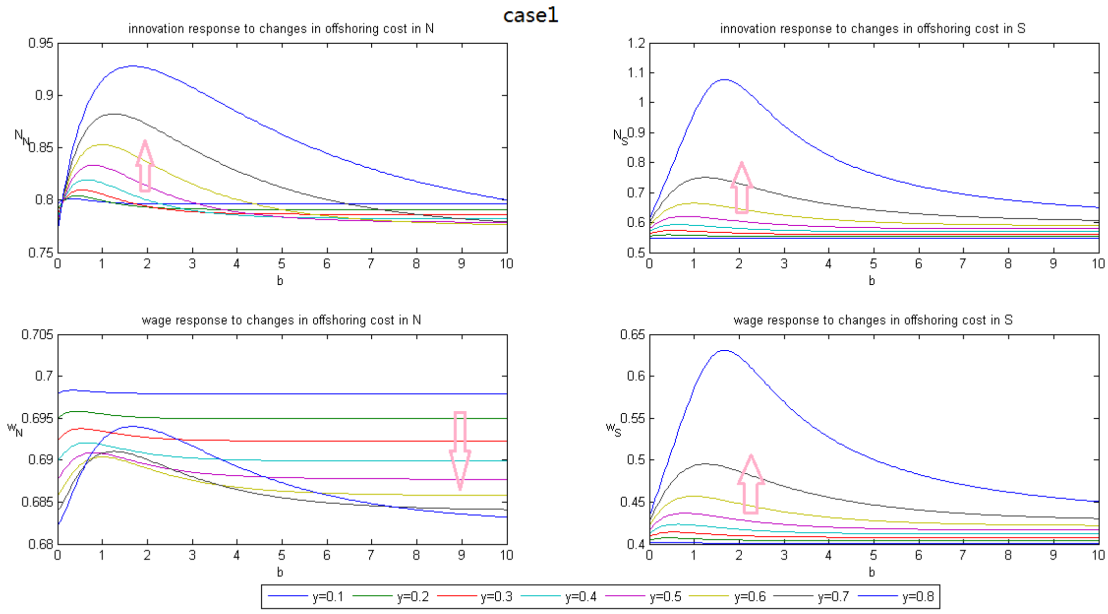
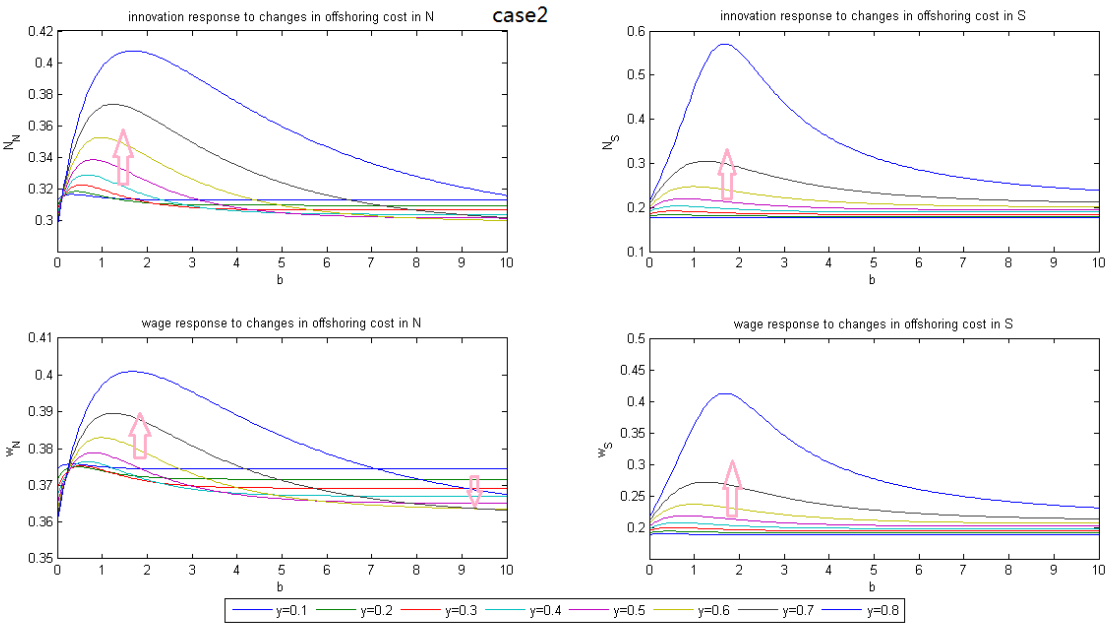
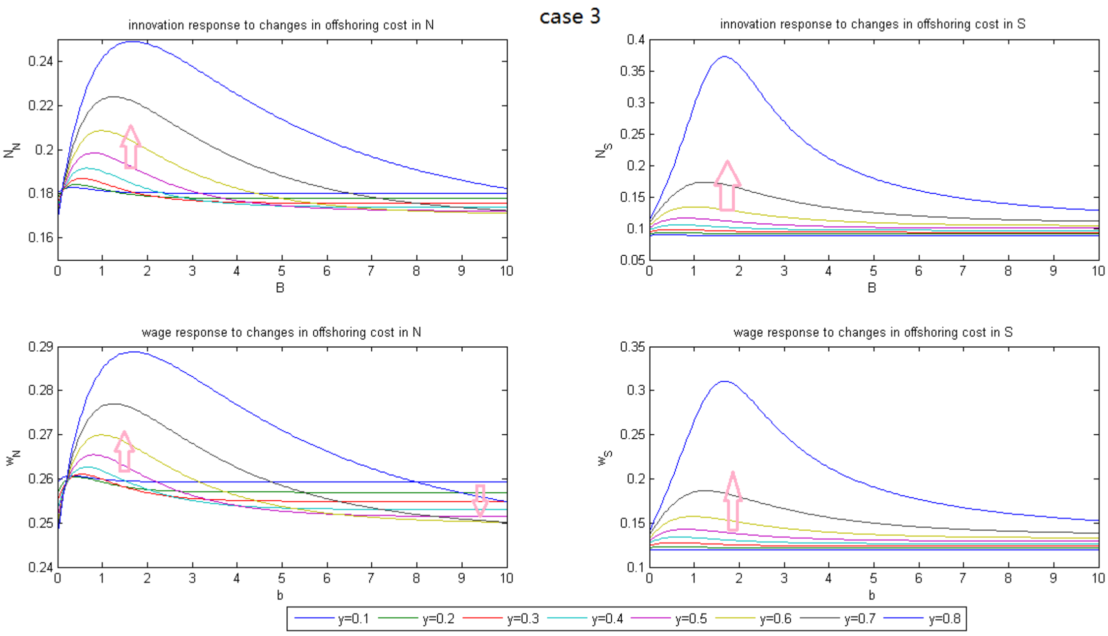
A Graphs in Section 4.2

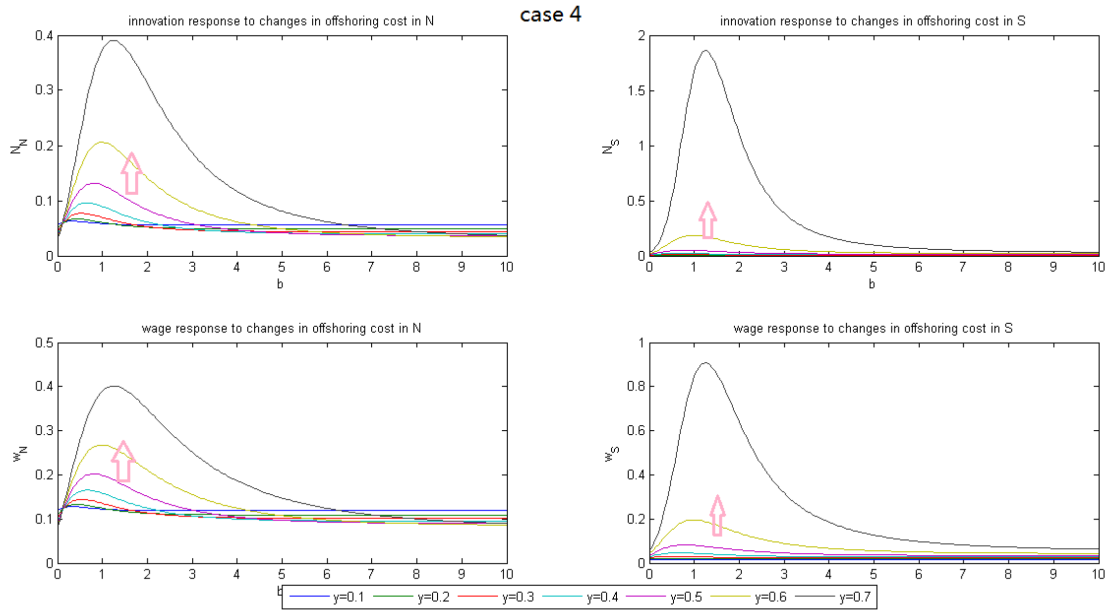
I present graphs of innovation response, wage response in each country and wage gap response to changes in offshoring cost in this section. I considered 10 different situations defined by the following parameters. Why I choose this 10 cases is discussed in section 4.2 in the paper.

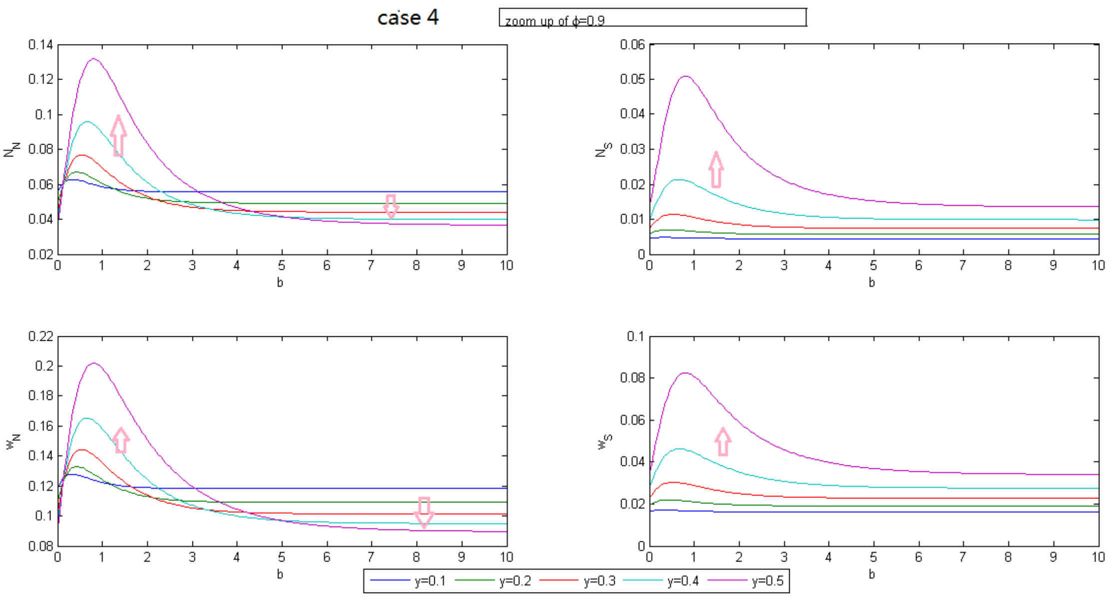
	ϕ	λ	γ	L_N	L_S
Case1	**-0.9**	0.9	0.4	1	1
Case2	**-0.1**	0.9	0.4	1	1
Case3	**0.1**	0.9	0.4	1	1
Case4	**0.9**	0.9	0.4	1	1
Case5	**1.5**	0.9	0.4	1	1
Case6	0.1	0.9	**0.8**	1	1
Case7	0.1	0.9	0.4	1	**2**
Case8	0.1	**1.7**	**1.2**	1	1
Case9	0.1	**1.7**	**1.5**	1	1
Case10	**1.5**	**1.7**	**1.2**	1	1


case1
innovation response to changes in offshoring cost in N
innovation response to changes in offshoring cost in S
wage response to changes in offshoring cost in N
wage response to changes in offshoring cost in S
b
y=0.1
y=0.2
y=0.3
y=0.4
y=0.5
y=0.6
y=0.7
y=0.8

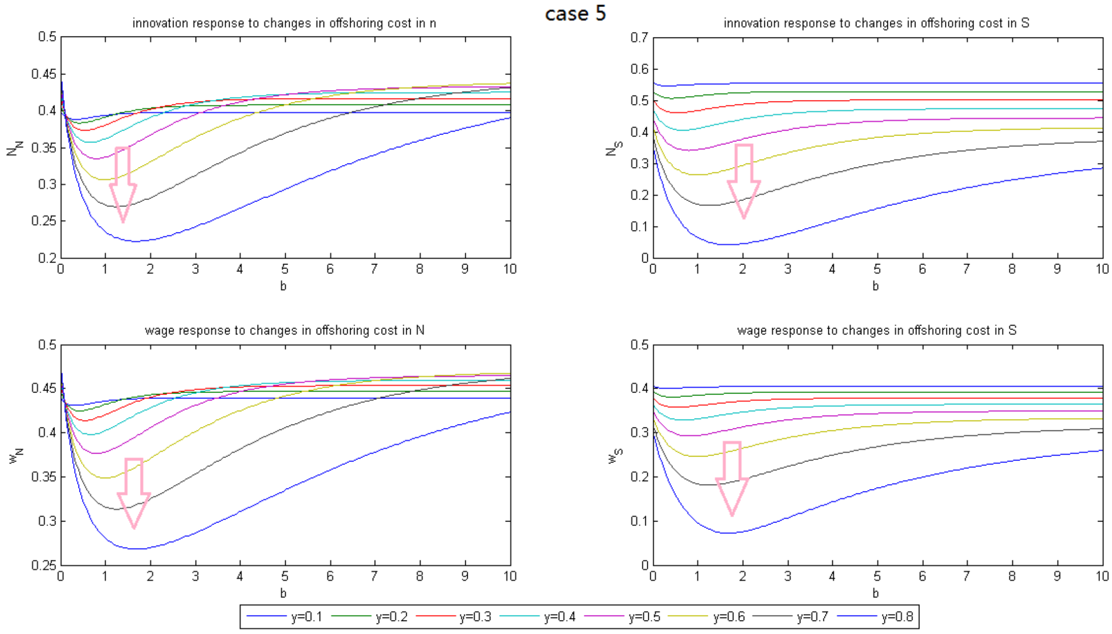

case2
innovation response to changes in offshoring cost in N
innovation response to changes in offshoring cost in S
wage response to changes in offshoring cost in N
wage response to changes in offshoring cost in S
b
y=0.1
y=0.2
y=0.3
y=0.4
y=0.5
y=0.6
y=0.7
y=0.8


case 3
innovation response to changes in offshoring cost in N
innovation response to changes in offshoring cost in S
wage response to changes in offshoring cost in N
wage response to changes in offshoring cost in S
N_N
N_S
w_N
w_S
B
b
y=0.1
y=0.2
y=0.3
y=0.4
y=0.5
y=0.6
y=0.7
y=0.8


case 4
innovation response to changes in offshoring cost in N
innovation response to changes in offshoring cost in S
wage response to changes in offshoring cost in N
wage response to changes in offshoring cost in S
y=0.1
y=0.2
y=0.3
y=0.4
y=0.5
y=0.6
y=0.7

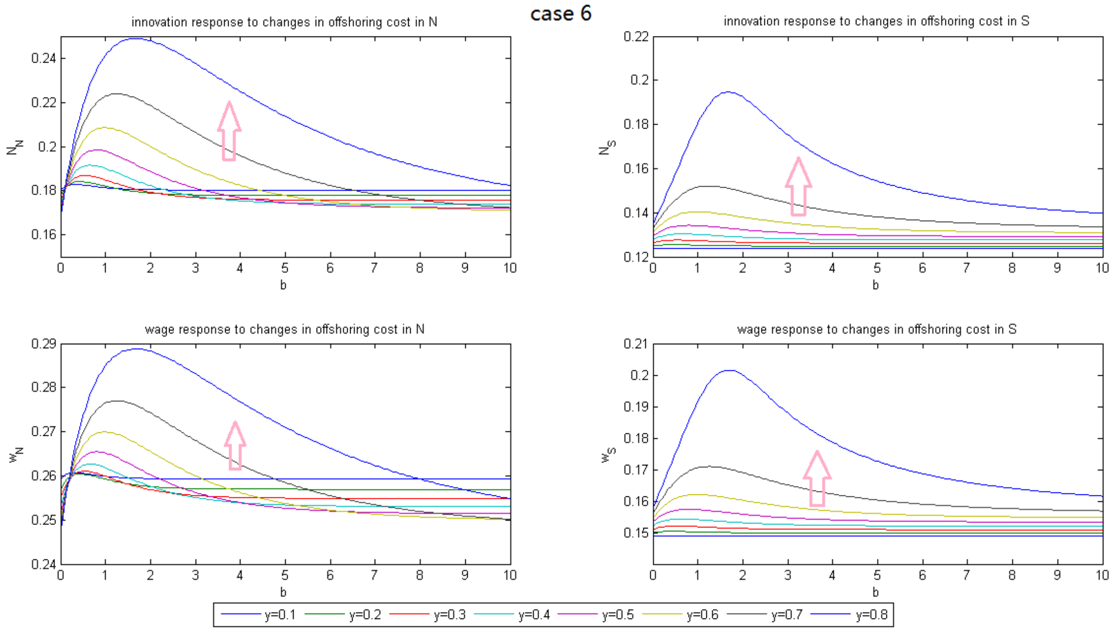

case 4
zoom up of ϕ=0.9
y=0.1
y=0.2
y=0.3
y=0.4
y=0.5

case 5



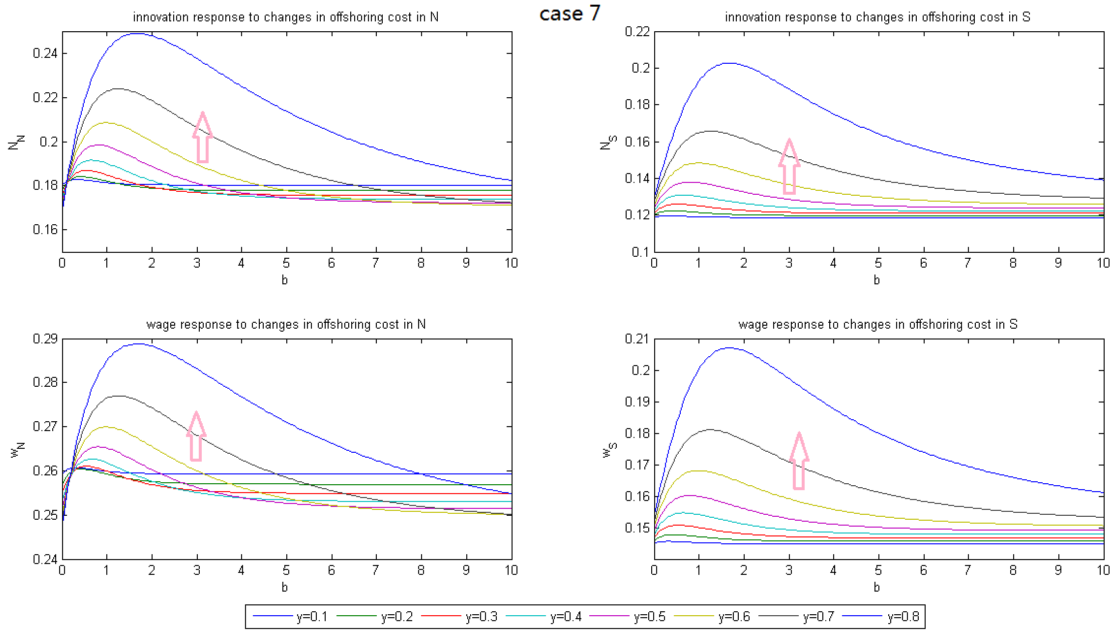
innovation response to changes in offshoring cost in n
innovation response to changes in offshoring cost in S
wage response to changes in offshoring cost in N
wage response to changes in offshoring cost in S
N_N
N_S
w_N
w_S
b
y=0.1
y=0.2
y=0.3
y=0.4
y=0.5
y=0.6
y=0.7
y=0.8

case 6

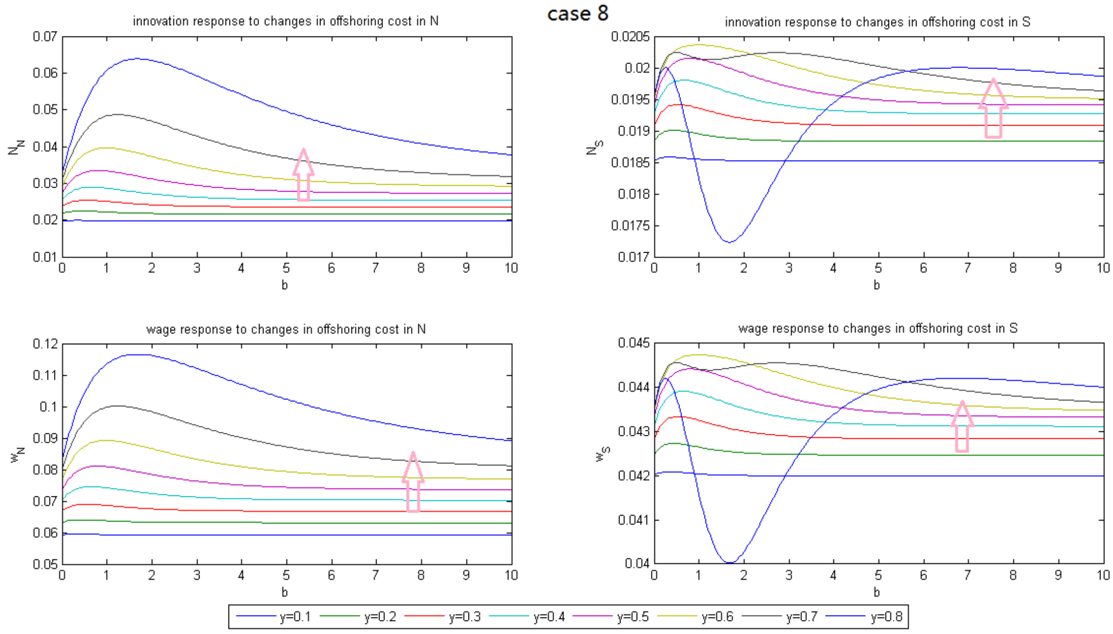


innovation response to changes in offshoring cost in N
innovation response to changes in offshoring cost in S
wage response to changes in offshoring cost in N
wage response to changes in offshoring cost in S
N_N
N_S
w_N
w_S
b
y=0.1
y=0.2
y=0.3
y=0.4
y=0.5
y=0.6
y=0.7
y=0.8

case 7


innovation response to changes in offshoring cost in N
innovation response to changes in offshoring cost in S
wage response to changes in offshoring cost in N
wage response to changes in offshoring cost in S
N_N
N_S
w_N
w_S
b
y=0.1
y=0.2
y=0.3
y=0.4
y=0.5
y=0.6
y=0.7
y=0.8

case 8


innovation response to changes in offshoring cost in N
innovation response to changes in offshoring cost in S
wage response to changes in offshoring cost in N
wage response to changes in offshoring cost in S
N_N
N_S
w_N
w_S
b
y=0.1
y=0.2
y=0.3
y=0.4
y=0.5
y=0.6
y=0.7
y=0.8

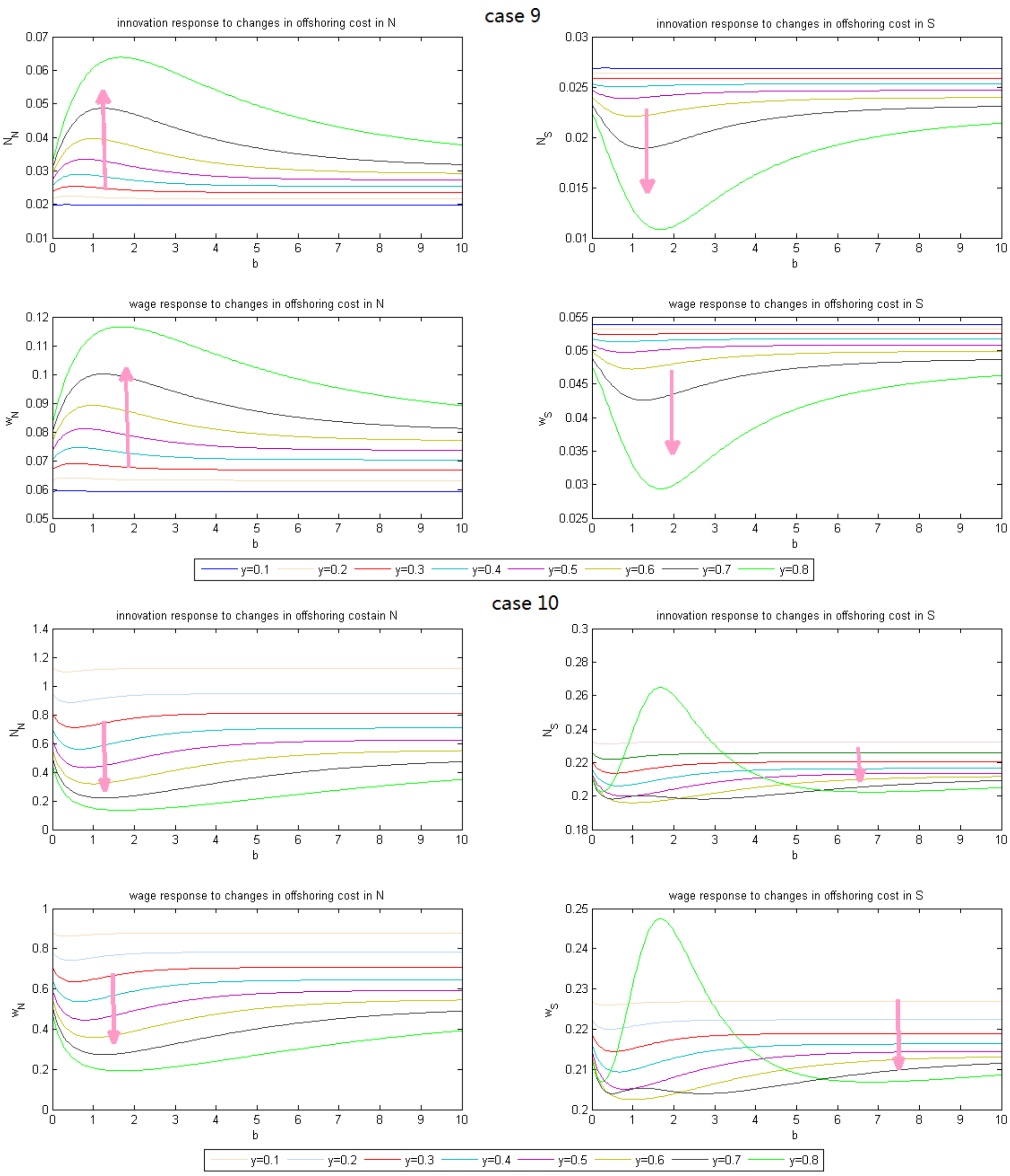
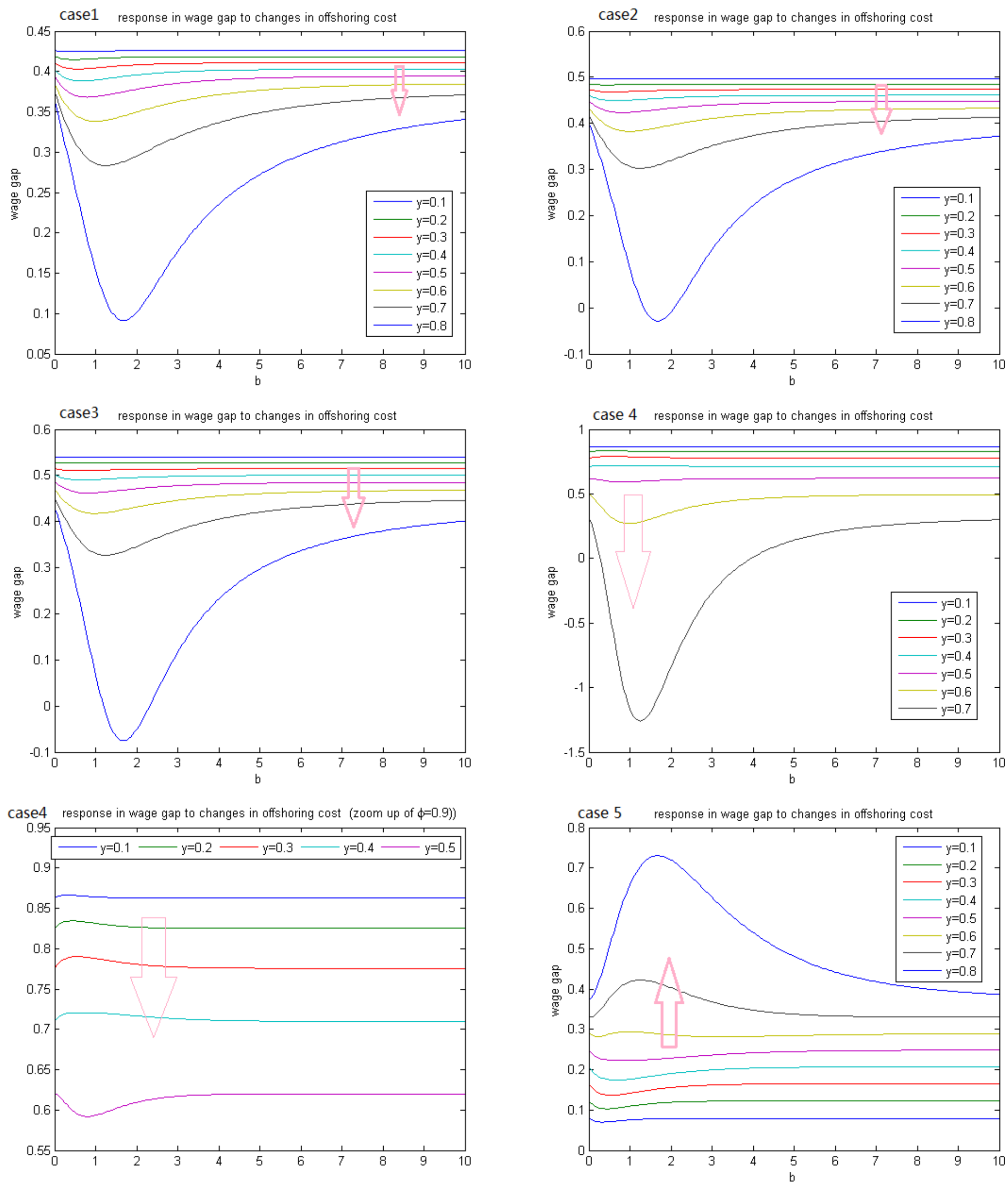


Figure 3: innovation and wage response in different cases. Allow shows the direction of y increase.


case1 response in wage gap to changes in offshoring cost
wage gap
b
y=0.1
y=0.2
y=0.3
y=0.4
y=0.5
y=0.6
y=0.7
y=0.8
case2 response in wage gap to changes in offshoring cost
wage gap
b
y=0.1
y=0.2
y=0.3
y=0.4
y=0.5
y=0.6
y=0.7
y=0.8
case3 response in wage gap to changes in offshoring cost
wage gap
b
case 4 response in wage gap to changes in offshoring cost
wage gap
b
y=0.1
y=0.2
y=0.3
y=0.4
y=0.5
y=0.6
y=0.7
case4 response in wage gap to changes in offshoring cost (zoom up of ϕ=0.9))
y=0.1
y=0.2
y=0.3
y=0.4
y=0.5
case 5 response in wage gap to changes in offshoring cost
wage gap
b
y=0.1
y=0.2
y=0.3
y=0.4
y=0.5
y=0.6
y=0.7
y=0.8

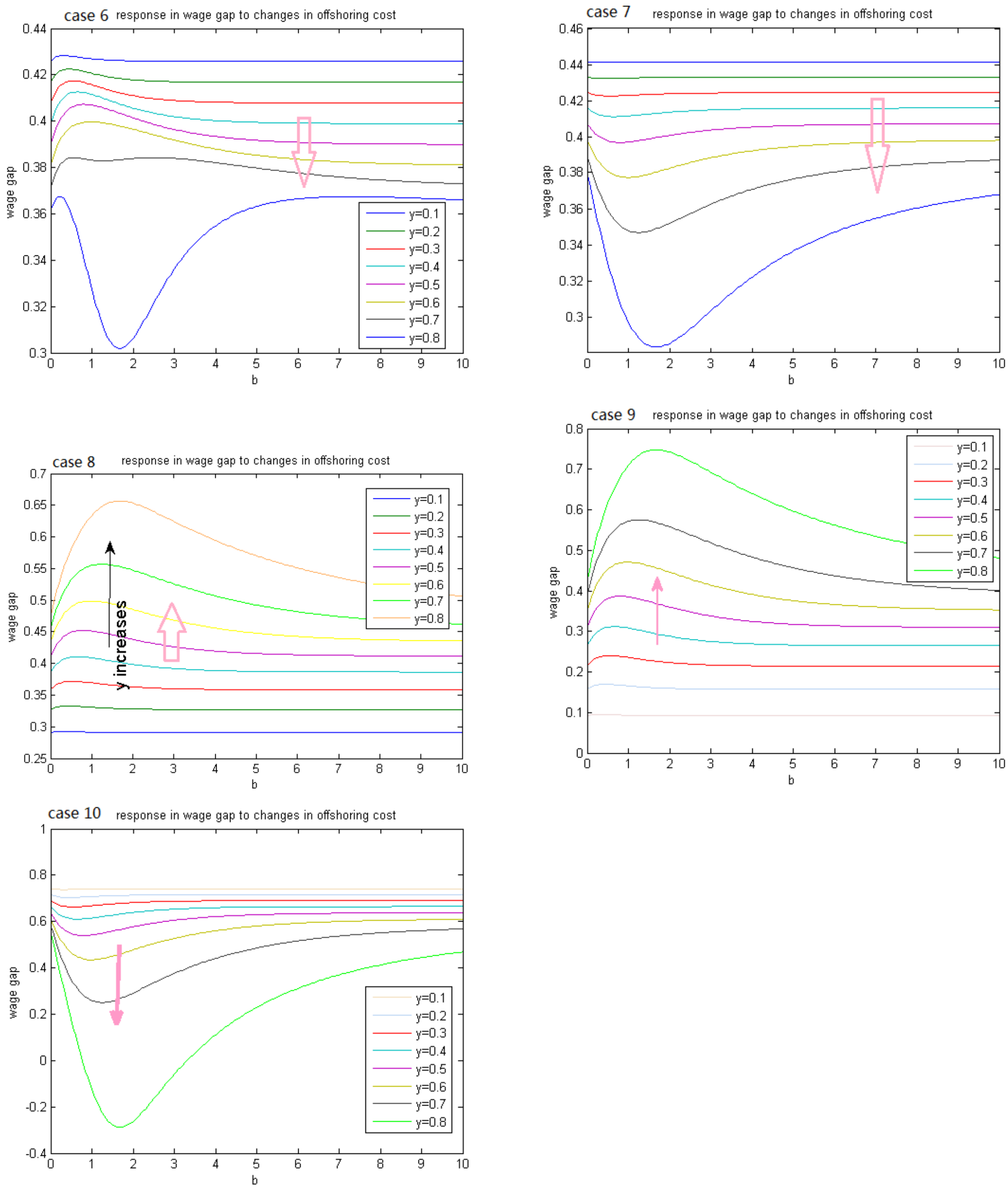


Figure 4: response of wage gap in different cases. Allow shows the direction of y increase.